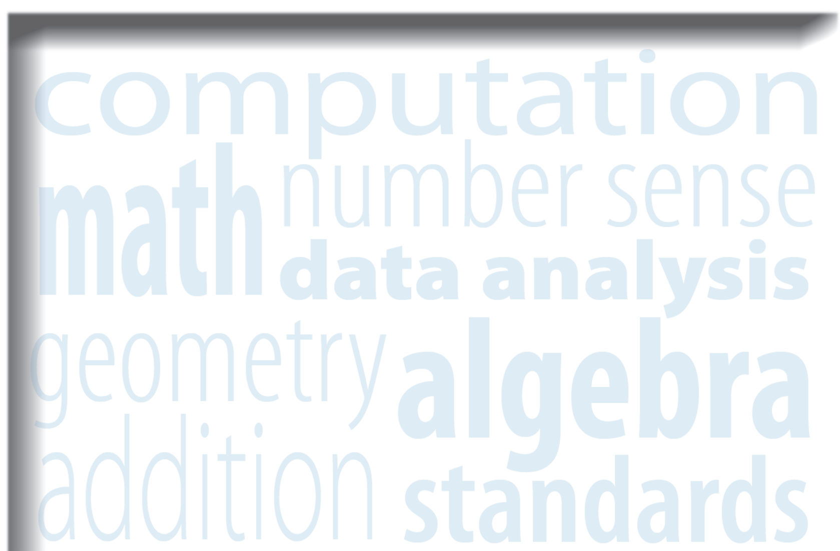
National Center on Education and the Economy

2008

International Benchmarking
Indiana's Academic Standards in Mathematics



Indiana • Singapore • Finland

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EXECUTIVE SUMMARY

With the emergence and growth of the global economy, many education policy makers have turned to international comparisons to help guide the design and development of national and state performance standards. Since the 1960s, the United States has participated actively in international projects designed to provide key information about the performance of its education system relative to systems in other countries. These comparisons shed light on a host of issues regarding the rigor, depth, coherence, and content of the mathematics students are learning. They let U.S. policy makers examine different aspects of countries' standards and curricula, assess these systems' performances, and identify potential strategies to improve student achievement within our own system.

This study compares Indiana's mathematics standards with those of Singapore and Finland, two countries whose students score consistently high on the international mathematics assessments.

Overall, Indiana fared well. The comparisons revealed high quality, rigorous mathematics taught in a good progression. The report also identifies areas where Singapore and Finland take different approaches, use a different timeline, or cover topics in more or less depth than the U.S. in general and Indiana in particular.

MAJOR FINDINGS

- ➜ Arithmetic operations are spread more evenly over grades K–6 in Singapore and Finland, where they are introduced earlier and more incrementally. Indiana students (and U.S. students in general) are introduced to multiplication and division later but then are expected to learn more per grade level in upper elementary to surpass the standards of their international counterparts by the end of 6th grade.

- ➜ The international emphasis on solving multi-step problems is consistent with promoting conceptual understanding through thoughtful problem solving.

- ➜ The international standards provide a coherent progression and solid foundation in the concept of quantity and units and in the relationships among quantities—addressing measurement in primary grades; with areas, rates, and variables in the upper elementary grades; and functions and graphs in middle school. This progression and emphasis builds a strong conceptual foundation that prepares students for work with the concepts of variable and function in middle school and algebra courses.

➜ To maximize efficiency from upper elementary mathematics through algebra, there should be a more explicit treatment of the application of the number properties and properties of equality in the standards to the progression of topics introduced in the upper elementary grades and middle school. These are major organizing ideas that form the foundation of the language of algebra used in arithmetic.

➜ Standards for high school in the U.S. must address three challenging policy issues. They should specify the mathematics content necessary for:

1. Students preparing to major in science, technology, engineering, or math (STEM).
2. Students preparing to major in humanities, business, or social science in college (non-STEM). This content should make students eligible for college without remediation.
3. Students preparing for all levels for work in a high-tech economy.

Standards for the traditional high school mathematics courses could be refined and improved. The most urgent need is to redesign the college preparatory sequence for non-STEM majors to equip them with more useful mathematics skills. Algebra II does not address the needs of non-STEM majors. Indiana's revised standards are a big improvement, but the pathways and options in the international systems offer more useful mathematics to more students, while still allowing university access to all.

➜ The presentation of the standards in the international benchmarking countries focuses concisely on the mathematical content, while emphasizing the metacognitive processes students should engage in. The documents focus on the "what" of the mathematics and on students' development of "habits of mind" in studying and communicating mathematics. In contrast, the U.S. tradition, as reflected in Indiana's draft, focuses on the kinds of problems students should be able to get answers for, regardless of the mathematics they use to get the answers.

Section A

OVERVIEW

Overview of Selected International Education Systems

A meaningful discussion of similarities and differences between Indiana's mathematics standards and those of Singapore or Finland requires a basic overview of the educational systems in which the mathematics is taught.

SINGAPORE

Singapore has devised a very successful system of mass education that is both free and universal. The public education system is highly centralized, with curriculum and standards that are uniform across all schools. National high-stakes assessment examinations measure achievement in about the 6th, 10th, and 12th year of education. The centralized authority—the Ministry of Education—is responsible for formulating and implementing educational policies, developing national curriculum frameworks and guidelines, and administrating national examinations in collaboration with the Cambridge General Certificate of Education. The educational system in Singapore is governed by the principle of meritocracy, and merit is measured largely through the national examination system.

- **Primary Education**

  Education begins at a young age in Singapore. Typically, children attend two to three years of "kindergarten" instruction, beginning as early as age 3. The kindergarten years are serviced privately, while compulsory public education begins at about age 5 or 6 when students enter "primary" school.

  At the completion of six years of primary education, students take the Primary School Leaving Examination (PSLE). This assesses students' achievement levels and determines their suitability for "secondary" education.

Singapore		United States	
Kindergarten, Levels 1–3	Approximate Age: 2–5	Kindergarten, Level 1	Approximate Age: 5
Primary, Levels 1–6	Approximate Age: 6–12	Elementary, Levels 1–6	Approximate Age: 6–12
Primary School Leaving Exam (PSLE)		No Exam	

The Singapore system recognizes that some students may need special assistance to attain competence. These students do not attain mastery by following Singapore's regular program of mathematics instruction, but follow an alternative course. Beginning in grades 5 and 6, Singapore identifies its weaker students on the basis of a an examination. These students are taught according to a special foundational 5th- and 6th-grade mathematics framework. Students receive special assistance by:

- more time—approximately 30 percent more mathematics instruction than students in the regular track, and
- exposure to the same mathematical content as students in the regular track, although at a slower pace.

Note: This report compares Singapore's *Mathematics Syllabus Primary* Levels 1–6 to Indiana's Grades 1–6 standards.

Secondary Education

Singapore's secondary level entails four to five years of education roughly equivalent to that of U.S. grades 7–10. Based on their PSLE exam performance, students enter one of four secondary streams: Special, Express, Normal Academic, or Normal Technical. Special Stream students take advanced language or "Higher Mother Tongue." After four years, both the Special and Express Streams take the Singapore-Cambridge General Certificate Exam, Level O (GCE 'O'). After four years, the Normal Academic Stream (NA) and the Normal Technical Stream (NT) take the Singapore-Cambridge General Certificate Exam, Level N (GCE 'N'). Normal Stream students who perform well in the GCE 'N' may continue with the program for a 5th year, moving on to take the GCE 'O'. Each secondary school offers all streams, and students are able to move from one stream to another, based on merit.

The four streams see a yearly distribution of students similar to that recorded in 2006:

Special Stream	9.1%
Express Stream	52.0%
Normal Academic Stream	24.5%
Normal Technical Stream	13.5%

Ministry of Education, 2006

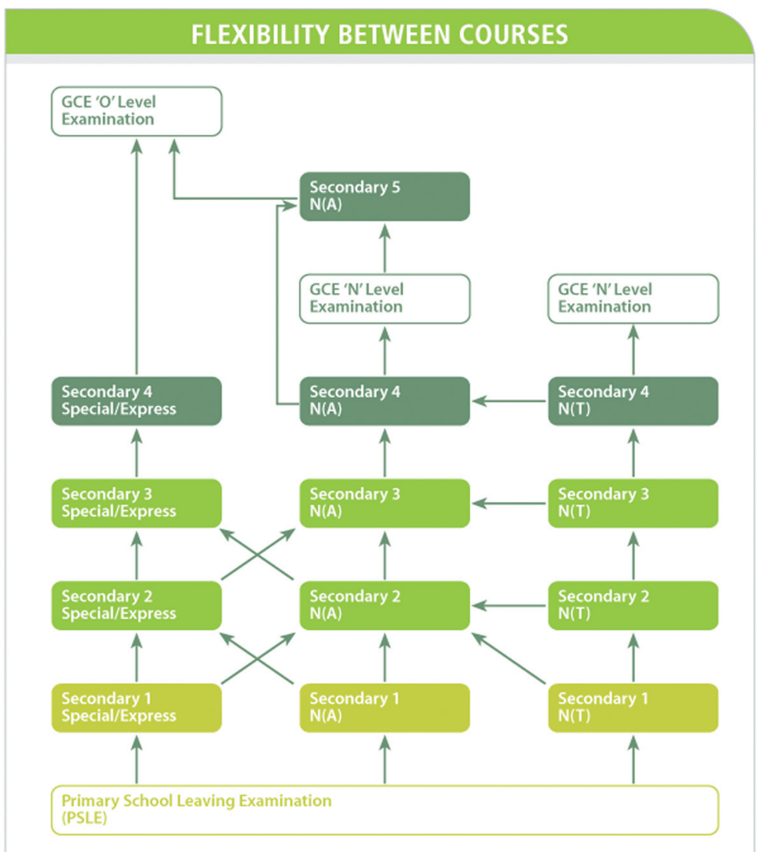
Singapore also offers an Integrated Programme (IP) which is designed for students who are self motivated, clearly university-bound, and ready to thrive in a less structured environment. The program spans secondary and pre-university education and does not include intermediate national examinations at the end of secondary school. In this program, time normally used to prepare students for the GCE 'O' is used to broaden students' learning experiences. The IP Programme leads to a Baccalaureate Diploma or A-level Exam.

Secondary Level Streams, Crossing Over

Candidates for the GCE 'O' are required to take certain core subjects, including English, mother tongue, elementary mathematics, combined humanities, science, and one additional subject. All four curriculum streams in the secondary level cover a range of core courses. The curriculum sequence provides built-in flexibility so that students who work hard and perform well in one stream can move upward to another. When stream changes are considered, students' performance records and personal assessments by the principal and/or classroom teachers are taken into account.

For instance, it is possible for a Normal Academic secondary student to move to the Express Stream after Year 2. Likewise, a Normal Technical student may choose to cross over to the Normal Academic Stream after Year 3, take the GCE 'N' after Year 4, then continue his or her education and finally sit for the GCE 'O' after the 5th year.

The chart below illustrates flexibility and opportunity for diagonal and upward movement within Singapore's secondary education system.



Ministry of Education, Singapore,
Nurturing Every Child, page 21

Note: This report compares Singapore's *Secondary Mathematics Syllabuses* Years 1 and 2 to Indiana's Mathematics Standards for grades 7–8. Secondary Years 3 and 4 comprise a portion of Singapore's syllabus that are compared to Indiana's high school standards.

- **Pre-University (or Post-Secondary) Education**

After 4 to 5 years of secondary study and successful completion of the GCE 'O' at age 16 or older, students are presented with a variety of options. For instance, they may pursue an academic course of study or a "professional centric" course of study that focuses on professional-level technical education. As determined by their GCE 'O' scores, individual students continue their education either in pre-university or post-secondary institutions.

Pre-University

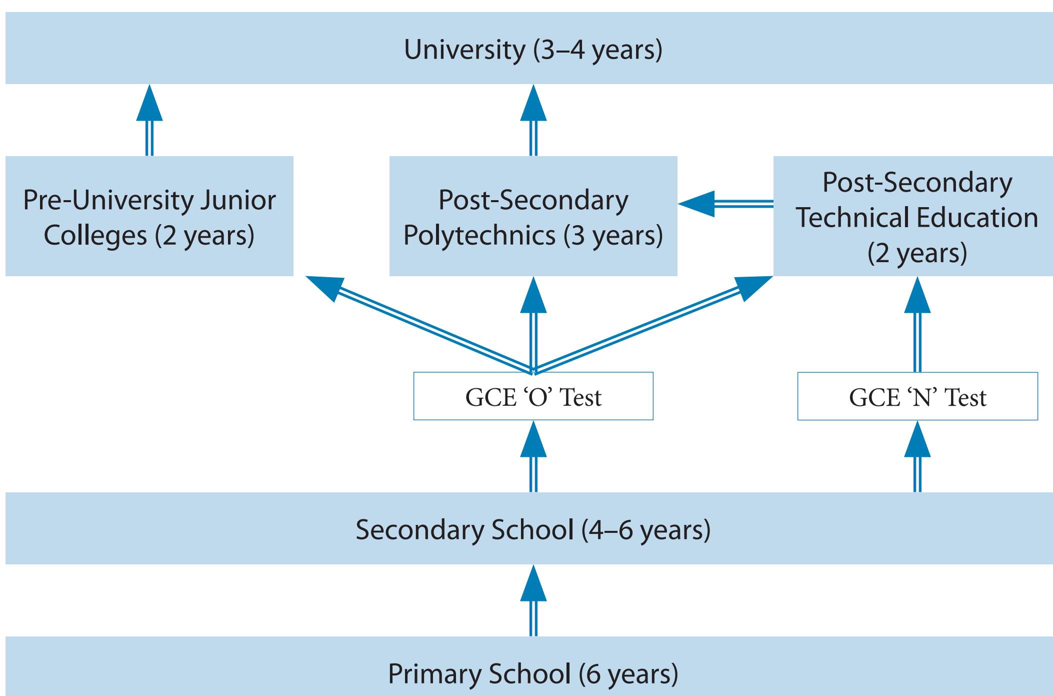
- Junior Colleges (JCs)
- Centralized Institute (CI) or the Millenia Institute

Post-Secondary

- Polytechnic Institutes
- Institute of Technical Education

Academically inclined students with the necessary GCE 'O' qualifications, will usually enter the pre-university system. Within the pre-university system there are different curricula. H1 and H2 level courses are complementary; H1 is typically half the breadth of an H2 course, although equal in depth. H3 courses are of the highest level and encourage critical thinking and may involve research.

Polytechnic Institutes offer 3-year diploma courses for students who wish to pursue applied and practice-oriented professional training. Students must also have the necessary GCE 'O' Level qualifications to enter a polytechnic school. Polytechnic courses are often specialized and may include specific fields, such as marine engineering, business management, digital communications, and the like. Polytechnic graduates with good grades also have the opportunity to pursue tertiary education at the universities.



Technical education is for students with GCE 'O' or 'N' Level certificates. Technical Institutes offer 1- to 2-year technical and vocational courses. Students who do well can go to polytechnics for a diploma program. Qualified candidates may also progress to the universities for subsequent study.

Regardless of the path taken—whether beginning in a Technical Institute or a Junior College—all students have the opportunity to apply themselves and work toward university-level studies. All students must take the GCE 'A' Level exam for entrance into the university.

Note: The H1 mathematics as defined by the H1 test—the first level of the pre-university mathematics courses—were used to compare to Indiana's high school standards, along with Secondary 3 and 4.

FINLAND

Finland's education system is based on providing all children and young people with equal basic education services. In Finland, education is compulsory, starting from the year the child becomes seven years old and ending when he or she reaches 16, a total of nine years of basics. Both municipal and private day-care services are available for children below the school age of seven. All 6-year-olds are entitled to at least one year of pre-school education before beginning their basic education. Pre-school education is available in school settings and in day-care centers.

- **Compulsory Basic Education (Ages 7–16)**

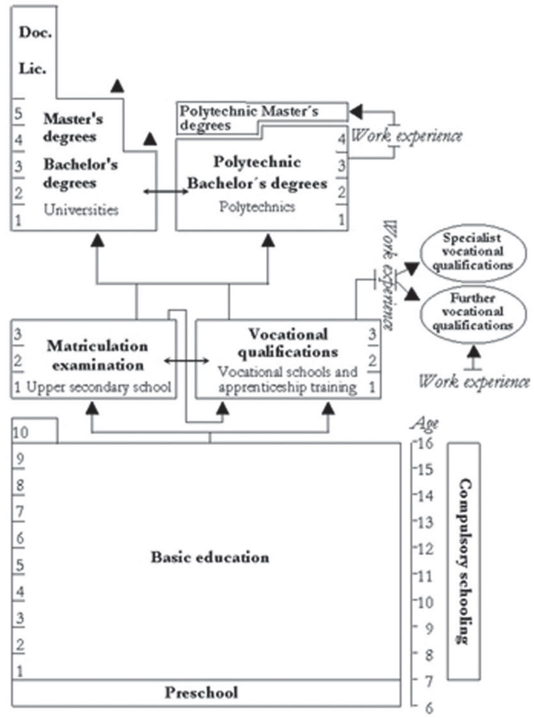
  The Finnish National Board of Education outlines the learning goals and defines the main content of basic education. The board sets the National Core Curriculum, the guidelines that govern all education providers. The National Core Curriculum defines not only the goals and content of the various subjects but also the cross-curricular themes. These themes are expected to be integrated in a child's upbringing and education. Cross-curricular themes are responsive to the educational challenges of the time. All education providers must draw up local municipality-specific (or school-specific) curricula as guided by the National Core Curriculum and any pertinent legislation.

  Note: Because the education of a child in Finland begins at age seven, making exact correlations to Indiana grade by grade becomes problematic. However, it is possible to concentrate on progression and sequence; these will be the determining factors when comparing the two systems. In this report, Indiana grades K–8 are compared with Finland grades 1–8.

- **Upper Secondary Schools**

  Education after primary school is divided into vocational and academic systems, according to the old German model. Traditionally, these two upper secondary systems did not inter-operate, although today some restrictions have been lifted.

  Approximately 92% of those who completed basic education in 2003 continued directly to upper secondary school. Typically, students enter either a trade school (vocational upper secondary) or an academic-oriented General Upper Secondary program. Trade school graduates may enter the workforce directly after graduation, while those in General Upper Secondary school do not study vocational skills, as they are expected to continue to tertiary education.



General Upper Secondary Education

The General Upper Secondary program is divided into courses. Each course consists of about 38 lessons. Therefore, year-long classes are not required. In addition, all general upper secondary schools are now non-graded.

The *National Core Curriculum for Upper Secondary School* requires a minimum of 75 courses. Of these, some are compulsory, some are considered specializations, and still others are deemed applied courses. Applied courses may be integrated courses combining elements from different subjects, various methodological courses, or vocational skills.

Matriculation

The final examination at the end of the General Upper Secondary program—the matriculation examination—is drawn up by a centralized body. This body validates the individual tests against uniform criteria. The matriculation examination consists of a series of at least four tests. Every student must take the test in his or her mother tongue. Students may select three other tests to take, chosen from possibilities including the second national language, a foreign language, mathematics, sciences, and humanities. In addition, students are free to take other optional tests.

Note: In this report, the Indiana high school standards were compared with compulsory courses in the *National Core Curriculum for Upper Secondary School.*

- **Decision Making**

Decisions regarding broad national objectives as well as the more specific aspects of education are all decided by the government. By devising the National Core Curriculum, the Finnish National Board of Education determines the objectives and core contents in the various subjects.

Beginning with the board's guidelines, education providers then draw up their own local curricula. The system must provide students with individual choices concerning studies, including the ability to utilize instruction given by other education providers, if necessary.

- **A Special Note Regarding Teacher Education**

When comparing the standards documents, it soon becomes evident that the *National Core Curriculum for Upper Secondary School* outlines Finland's mathematics objectives with a rather broad stroke and much less specificity than do Indiana's Mathematics Standards. The National Board of Education recognizes the high caliber of Finland's teachers, most of whom possess an advanced understanding of core concepts in mathematics and mathematics instruction. Hence, the document reflects teachers' level of mathematical competency.

This high competency is developed within Finland's teacher education system. Finland's Teacher Education system has been identified as one of the top training programs in the world. (Barber and Mourshed, 2007) Finland begins by selecting only top candidates for their university system. Descriptive snippets of the selection process follow:

> "… multiple-choice examinations designed to test numeracy, literacy and problem-solving skills… top scoring candidates are then passed through a second round in the selection process that is run by individual universities… applicants are tested for their communications skills, willingness to learn, academic ability and motivation for teaching." (Barber and Mourshed, 2007)

In Finland, the teaching profession is a competitive field. Even after intensive preparation and successful completion of the training program, graduates apply to individual schools where they must score well on additional exams in order to win a position. Young men and women work hard to gain admission to Finland's Teacher Education system. And they continue to work hard in a culture that places high value on continual growth and improved practice, where "professional development is regarded as essential and is organized extensively." (NBE, 2008)

Finland's teachers possess a level of competency that is reflected in the spare and open guidelines provided in the National Core Curriculum. Their high degree of professionalism affords them leadership opportunities and a great deal of local control, not to mention instructional freedom.

> "teachers are considered pedagogical experts, and are entrusted with considerable independence in the classroom, and also have decision-making authority as concerns school policy and management. They are deeply involved in drafting the local curricula and in development work. Furthermore, they have almost exclusive responsibility for the choice of textbooks and teaching methods." (NBE, 2008)

Singapore has a similar screening system which is used to test and select applicants before they enter teacher education at the university level. This results in highly-trained teachers. Even so, the Singapore Ministry of Education is more directive in its syllabi.

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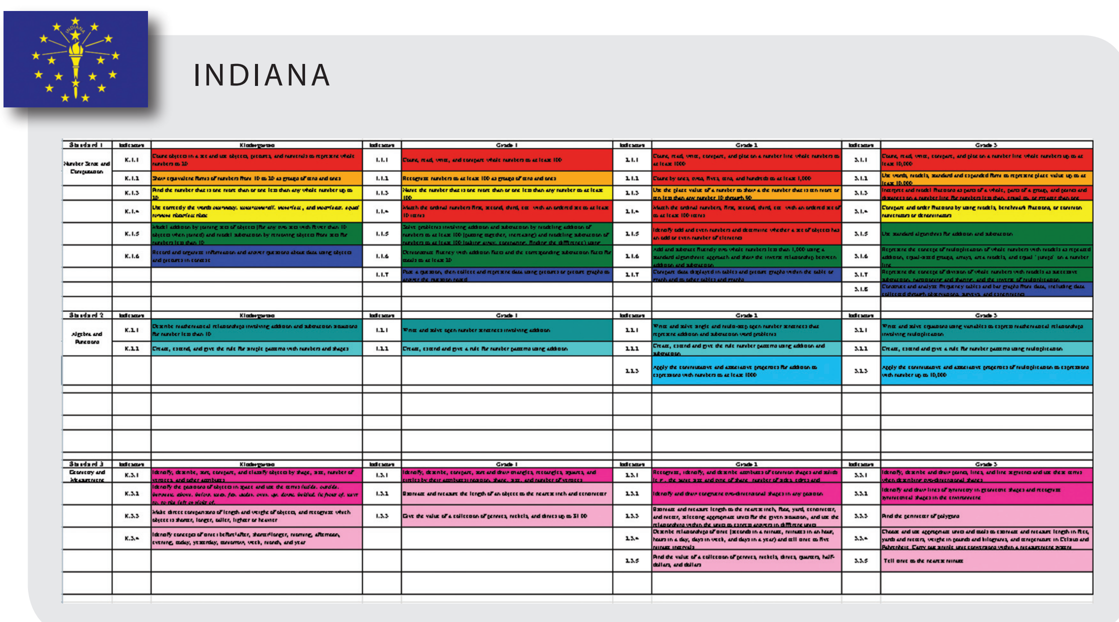
The Features of the Standards

Indiana, Finland, and Singapore describe the requirements of its mathematics programs in different ways; however, similarities can be found. First, Singapore calls its document a syllabus, rather than a "set of standards." This report looks at the Singapore *Mathematics Syllabus Primary* and the Singapore *Secondary Mathematics Syllabuses* plus the H1 test items used in the pre-university system. The term "standards" does not appear in Finland's title either—its document is referred to as the National Core Curriculum. In this report, both the *National Core Curriculum for Basic Education* and the *National Core Curriculum for General Upper Secondary* were used.

When readers view the documents side by side, one of the first impressions they might notice is the "look" of the standards—that is, the format in which they are presented. When the *Indiana Standards for Mathematics* are compared to Singapore's *Mathematics Syllabus Primary* and Finland's *National Core Curriculum for Basic Education*, several differences become evident. These differences reflect not only the clarity of the presentations but the teachers' assumed level of understanding of the mathematics. Some of the distinguishing features are discussed below.

- **Concepts across Grade Levels**

  One way Indiana represents its standards is on a multi-color chart, showing how concepts—place value, comparing and ordering numbers, operations, and the rules of arithmetic, for instance—appear across the grade levels. The specific mathematical concepts are a major organizing factor in the presentation of the standards.



  This approach allows teachers to see how students' understanding of a concept builds from one year to the next by highlighting the similar concepts that appear a year later or earlier.

- **Use of Examples**

The *Indiana Standards for Mathematics* also include examples. Examples remove ambiguity so that teachers can determine exactly what is meant by the standard.

INDIANA

Number Sense and Computation Grade 1

1.1.2 Recognize numbers to at least 100 as groups of tens and ones.

Example:

Use base ten blocks to model 34 using 3 longs (tens) and 4 units of one; then using 34 units of one; and finally, using 2 longs (tens) and 14 ones.

- **Limits—What Is Not Covered**

As expected, the Singapore *Mathematics Syllabus Primary* guides teachers through the mathematics to be covered in the primary and secondary years. What is especially noteworthy about the portion of the syllabus shown below is that it goes a step further and describes what is *not* covered in a given year.

SINGAPORE

Multiplication and Division Primary 1

Include:

- multiplication as repeated addition (within 40)
- use of the multiplication symbol (×) to write a mathematical statement for a given situation
- division of a quantity (not greater than 20) into equal sets:
  - given the number of objects in each set
  - given the number of sets
- solving 1-step word problems with pictorial representation

Exclude:

- use of multiplication tables
- use of the division symbol (÷)

By stating, when necessary, the work that should be *excluded* from instruction, the Singapore syllabus enables teachers to focus their efforts and hone in on specific aspects of the concepts. The Singapore syllabus makes clear what students are responsible for knowing and when they are expected to know it. This *exclusionary clarity* serves to organize teachers' instruction around concrete, conceptual work so that students deepen their understanding of concepts at a reasonable pace. This also helps prevent the negative effects that can result when students are rushed through material that is beyond their level of comprehension.

- **Descriptions of Summary Expectations**

The Finland *National Core Curriculum for Basic Education* provides examples of what good student performances should be at the end of years 3, 5, and 8.

FINLAND

Description of Good Performance At End of Grade 5

Thinking and working skills

Pupils will

- demonstrate an understanding of concepts associated with mathematics by using them in problem-solving, and by presenting them in diverse ways—with instruments, pictures, symbols, words, numbers, or diagrams
- try consciously to focus their attention when making observations; they will be able to communicate their observations and thoughts in diverse ways—by acting, speaking, writing, and using symbols
- know how to depict real-world situations and phenomena mathematically by comparing, classifying, organizing, constructing, and modeling
- know how to group or classify on the basis of a given or chosen criterion, to look for a shared attribute, to distinguish between a qualitative and quantitative property, and to describe groups of things and objects, positing true and untrue propositions about them
- know how to present mathematical problems in a new form; they will be able to interpret a simple text, illustration, or event and to make a plan for solving the problem
- know how to follow rules.

Numbers, calculations, and algebra

The pupils will

- understand the decimal system in terms of decimal fractions, too, and know how to use it confidently; they will understand the concepts of the negative number and fraction and be able to present them by different methods
- know how to present calculations in writing and orally, and know the relationships between different calculations; they will know how to estimate in advance the magnitude of the result and, after the problem is solved, to check the stages of the calculation and evaluate the sensibleness of the solution
- know how to formulate and continue number sequences and to present correlations.

(continues)

The example above provides a summary description of what students should know and be able to do after five years of basic education. This offers teachers the big picture of where students should be headed

- **Use of Verbs: Do Verbs versus What**

In the U.S., mathematics standards are often written in a style that focuses on how students will do tasks rather than on what mathematics content they will learn. For example, U.S. math standards use a limited set of verbs to begin each standard. These words describe what students are expected to do: identify, recognize, measure, find, or solve.

Singapore and Finland stress the content of their programs with simple, concrete descriptions of the mathematics. Because the focus is on content, the work to be done is naturally revealed. Note the two examples from the Singapore *Mathematics Syllabus Primary* shown below.

SINGAPORE

Ratio Primary 6

Include:

- expressing one quantity as a fraction of another, given their ratio, and vice versa
- finding how many times one quantity is as large as another, given their ratio, and vice versa
- expressing one quantity as a fraction of another, given the two quantities,
- finding the whole/one part when a whole is divided into parts in a given ratio
- solving word problems involving 2 pairs of ratios

SINGAPORE

Ratio, Rate and Proportion Secondary 1

Include:

- ratios involving rational numbers
- writing a ratio in its simplest form
- average rate
- problems involving ratio and rate

On the same topic of ratio, the *Indiana Mathematics Standards* (see next page) defines the ratio standard—what the student needs to do—while the example helps to illustrate the mathematics content.

Indiana has the "do" statements but with the added example illustrating the mathematics; the "what" becomes more transparent.

INDIANA

Number Sense and Computation Grade 6

6.1.7 Interpret ratios, model ratios, and use ratios to show the relative sizes of two quantities. Use the notations: a / b, a to b, and $a : b$.

Example:

A car moving at a constant speed travels 130 miles in 2 hours. Write the ratio of distance to time and use it to find how far the car will travel in 5 hours.

6.1.8 Recognize proportional relationships and solve problems involving proportional relationships.

Example

Sam made 8 out of 24 free throws. Use a proportion to show how many free throws Sam would probably make out of 60 attempts.

6.1.9 Solve simple ratio and rate problems using multiplication and division, including problems involving discounts at sales, interest earned, and tips.

Example:

In a sale, everything is reduced by 20%. Find the sale price of a shirt whose pre-sale price was $30.

Notice each of the Indiana standards focuses on finding a single number. It is important to remember that there is a significant drawback to focusing too much on the "do." The real crux of the standard is the core mathematics. Do verbs can overshadow the core—the "what" of mathematics. When this occurs, the mathematics becomes fragmented and easily subordinated to teaching students how to get answers, without necessarily deepening their understanding.

In addition to the example problems shown in the above chart, some example questions could also be included. The right questions can extend students' learning and deepen their understanding. For example, in the free throw problem illustrating standard 6.1.8, the following questions would enrich the example:

- What is proportional to what?
- What rates can you calculate?
- What do the rates tell you?
- If Sam made 2 of his next 3 free throws, what would happen to the rate?

Questions like these require more of students than simply calculating answers; they require students to think deeply about what the core mathematics entails.

- **Use of Calculators**

  The Singapore syllabus provides substantial guidance on the use of calculators. In grades 5 and 6, the syllabus states that calculator use is allowed unless otherwise stated. When it is prohibited, specific standards or student activities will include the phrase "without using calculators."

  For instance, the example below assures teachers that in grade 5 students are expected to add and subtract proper fractions on their own, while it is acceptable for them to use calculators to add and subtract mixed numbers.

SINGAPORE

Fractions: Four Operations Primary 5

(Calculator is allowed unless otherwise stated.)

Include:

- addition and subtraction of proper fractions **without using calculators**,
- addition and subtraction of mixed numbers,
- multiplication of a proper fractions and a proper/ improper fraction **without using calculators**,
- multiplication of an improper fraction and an improper fraction,
- multiplication of a mixed number and a whole number,
- division of a proper fraction by a whole number **without using calculators**,
- solving word problems involving the 4 operations.

Summary of the Three Documents

The three documents, Singapore's syllabi (*Mathematics Syllabus Primary* and *Secondary Mathematics Syllabuses*), Finland's National Core Curriculum (*Basic Education* and *General Upper Secondary*), and Indiana's Mathematics Standards (K–8 and Secondary) all have unique features—the features being useful in different ways. Each document takes a unique approach to clarifying mathematical concepts, instructional sequences, and expected student performances at the different grade levels.

The Process Standards

Singapore, Finland, and Indiana all make somewhat comparable references to overall processes in their standards documents. Process standards refer to the major components that are critical to both the effectiveness of a mathematics program and to a student's ability to comprehend and fully engage with the mathematics. Each document treats the subject differently, but there are enough similarities that a discussion of the standards is merited.

INDIANA

The Indiana mathematics standards for K–8 divide process standards into five categories.

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

Three additional categories should be addressed at all grade levels in mathematics:

- Estimation
- Mental Computation
- Technology

SINGAPORE

The Singapore syllabus consists of several sections which correlate to Indiana's process standards. These appear in the "Aims of Mathematics Education in Schools" and "Mathematics Framework" sections of the *Mathematics Syllabus Primary* and the *Secondary Mathematics Syllabuses*.

- **Aims of Mathematics Education in School**

  This section offers broad statements regarding the aims of the mathematics program. It answers the question, "What is the intent of mathematics learning and what will it enable students to do and achieve?"

MATHEMATICS SYLLABUS PRIMARY

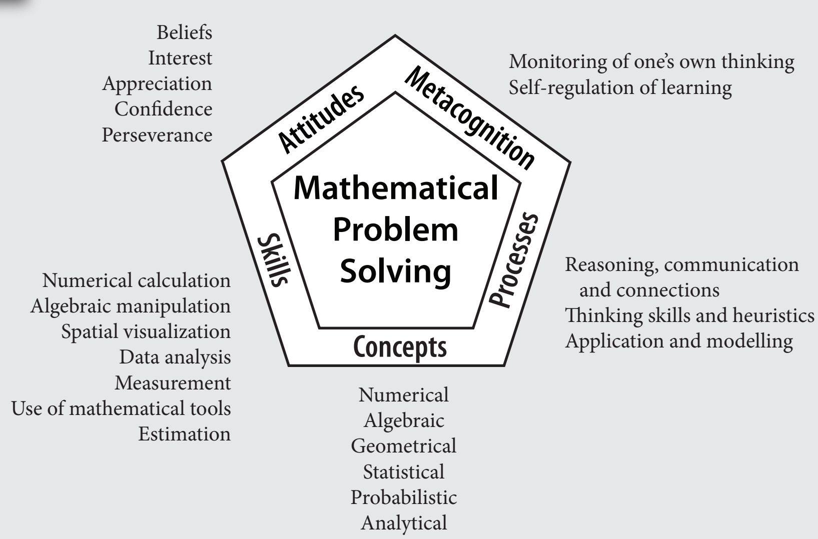
Mathematics education aims to enable students to:

1. Acquire the necessary mathematical concepts and skills for everyday life, and for continuous learning in mathematics and related disciplines.
2. Develop the necessary process skills for the acquisition and application of mathematical concepts and skills.
3. Develop the mathematical thinking and problem solving skills and apply these skills to formulate and solve problems.
4. Recognize and use connections among mathematical ideas, and between mathematics and other disciplines.
5. Develop positive attitudes towards mathematics.
6. Make effective use of a variety of mathematical tools (including information and communication technology tools) in the learning and application of mathematics.
7. Produce imaginative and creative work arising from mathematical ideas.
8. Develop the abilities to reason logically, communicate mathematically, and learn cooperatively and independently.

- **Mathematics Framework**

This section describes the importance of learning attitudes, metacognition, critical thinking, mathematical concepts, and required skills that are important components to achieving success in mathematics. (The Mathematics Syllabus Primary includes a section in Part B titled "Use of Calculator and Technology.") The section describes the underlying, principle components that frame an effective mathematics program: attitudes, metacognition, processes, concepts, and skills.

MATHEMATICS SYLLABUS PRIMARY



FINLAND

Each of the Finland grade spans, 1–2, 3–5, and 6–9, have summary statements which describe the objectives for that grade span.

FINLAND Grades 1–2

Summary Statement

The core tasks of mathematics instruction in the first and second grades are the development of mathematical thinking; practice concentrating, listening and communicating; and acquisition of experience as a basis for the formulation of mathematical concepts and structures.

Objectives

The pupil will:

- learn to concentrate, listen, communicate, and develop their thinking they will derive satisfaction and pleasure from understanding and solving problems
- gain diverse experience with different ways of presenting mathematical concepts; in the concept formation process, the central aspects will be spoken and written language, tools, and symbols
- understand that concepts form structures
- understand the concept of the natural number and learn the basic computational skills appropriate to it
- learn to justify their solutions and conclusions by means of pictures and concrete models and tools, in writing or orally; and to find similarities, differences, regularities, and cause-and-effect relationships between phenomena
- become practiced in making observations about mathematical problems that come up and are challenging and important from their personal standpoints.

DIFFERENCES AND SIMILARITIES IN PROCESS STANDARDS

- **Problem Solving**

The *Indiana Standards for Mathematics* lists problem solving as a "process" standard. Singapore has placed "mathematical problem solving" in the center of its mathematics framework. Singapore's philosophy is that mathematical problem solving is central to mathematics learning. The development of mathematical problem solving abilities is dependent on five inter-related components titled *Concepts*, *Skills*, *Processes*, *Attitudes* and *Metacognition*.

Finland also deals with "problem solving" but stresses becoming "practiced in making observations about mathematical problems that come up" and the ability to "find similarities, differences, regularities, and cause-and-effect relationships between phenomena."

(Also see the "Metacognition" section on on page 23.)

- **Reasoning and Proof**

  All three documents deal with the student's ability to reason. Indiana has specific bullet points which deal with proof and mathematical conjectures. Singapore discusses "habits of mind"—i.e., a student's ability to analyze mathematical situations and construct logical arguments. Finland states that students must be able to "justify" their actions, conclusions, and solutions and, in the higher grades, be able to "think logically" and creatively. Both Singapore and Finland emphasize justification over proof.

- **Communication**

  Again, all three documents emphasize the student's ability to communicate. Indiana requires students to organize their thinking, "communicate coherently to peers using the language of mathematics," and to evaluate the thinking of others. Singapore references similar objectives. Finland puts a great deal of emphasis on the students' ability to "present their thoughts" and ideas and to develop listening skills.

- **Connections**

  Indiana and Singapore reference the importance of "connections" in a similar manner. Finland does not discuss connections per se, but does include statements such as "find similarities, differences, regularities, and cause-and-result relationships between phenomena."

- **Representations**

  Representation is mentioned in all three of the standards documents. Singapore includes a section on thinking skills and heuristics which discusses thinking processes such as classifying, comparing, sequencing, etc. This section also discusses creating "a representation," making a calculated guess, implementing a process, or changing a problem. Finland includes statements like "gain diverse experience with different ways of presenting mathematical concepts; in the concept formation process, the central aspects will be spoken and written language, tools, and symbols."

- **Estimation and Mental Computation**

  Indiana discusses estimation and mental computation in its introductory materials. Singapore and Finland emphasize these skills within their descriptions of content.

- **Technology**

  Indiana and Singapore make statements on technology, whereas Finland does not. Singapore also emphasizes technology within its syllabus—see page 18 of this report. Singapore does not introduce technology until primary 5, while Indiana does not specify when it is introduced. Both agree that technology should not detract from the importance of mental and manual calculations.

- **Attitudes**

  Both Singapore and Finland emphasize the importance of student attitudes toward mathematics. Singapore includes “attitudes” as an element of its mathematics framework and also devotes a section of the introductory materials to the discussion of attitudes. In every grade level overview, Finland outlines objectives that involve attitudes—“student will derive satisfaction and pleasure” from understanding and solving problems (1–2); “gain experience in succeeding” with mathematics (3–5); and “learn to trust themselves, and to take responsibility” for their own learning in mathematics (6–9).

- **Metacognition**

  Singapore includes mention of metacognition—or “thinking about thinking.” The discussion stresses that students need to be able to “monitor” their own thinking. It provides some techniques that teachers can use to develop a student’s “metacognitive awareness” and to develop problem solving abilities. Finland places emphasis on students “developing their own thinking”—not just solving problems.

- **Applications and Modeling**

  Singapore discusses how applications and modeling play a vital role in the development of mathematical understanding and competencies. Finland deals with modeling in grade 3 and above with statements such as “introduce the learning of ‘mathematical models’ of thinking.”

K–8 STANDARDS

The K–8 mathematics standards of Indiana were compared with those of Singapore and Finland, based on coherence, progression, rigor, focus, and clarity. The following questions were answered:

- Do Indiana's draft standards address the most critical mathematics knowledge and skills, and do they represent the coherence of the discipline from an international perspective?
- Do the draft standards show a solid progression of content and skills from grade to grade, and/or course to course, and is the content grade-appropriate when compared to other nations?
- Do the draft standards represent a level of rigor characteristic of the exemplary K–6 and secondary mathematics benchmarks?
- Are the draft standards appropriately focused, demonstrating that choices have been made about what is most important for students to learn as represented by the international comparison?
- Are the draft standards written in specific, clear and measurable language?

Major differences between the *Indiana Standards for Mathematics* and those of Singapore and/or Finland are presented and discussed within each of the four Indiana Core Standards: Number Sense and Computation; Algebra and Functions; Geometry and Measurement; and Data and Probability. Within each Core Standard, the differences are discussed in order starting with cross grade-level, then the development of a particular idea, and finally order of concepts. Additionally, several mathematics educators have read the document and suggested some wording changes, which are included under "Additional Considerations" at the end of each Core Standard section.

Standard 1: **Number Sense and Computation**

- *Assets*

The *Indiana Standards for Mathematics* describe comparing and ordering skills that are more specific and give a greater number of examples than is given in corresponding standards from Finland or Singapore. For example, where Singapore states only "compare and order numbers," and Finland states "properties of numbers: comparison, classification, ordering…" Indiana states "Name the number that is one more than or one less than any number to at least 100." Comparing and ordering is developed through rational numbers and common irrational numbers including scientific notation, thus reinforcing the structure of the number system.

The *Indiana Standards for Mathematics* name the four arithmetic operations, collectively, as a sub-category of *Number Sense and Computation* in grades K–8. This promotes a mathematically useful view of the arithmetic operations as coherent across number representations. Too often, students learn the procedural rules of operating on whole numbers, fractions, and decimals and focus only on the surface distinctions among the different forms of number. Students who have spent some time attending to the fundamental arithmetic rules that characterize operations across number forms have a real advantage when, as they progress in algebra, the numbers disappear and are replaced by variables, demanding an understanding of the deeper rules of arithmetic.

Indiana's development of operations with whole number is well done—first starting with models, using number lines, then focusing on automaticity.

In general, the topics covered in Number Sense and Computation in the *Indiana Standards for Mathematics* are equivalent to the content covered in Singapore and Finland. The topics build from grade level to grade level and follow a spiral approach.

- *Difference* — **Development of the four operations**

In Indiana, as is generally the case throughout the United States, multiplication and division are not introduced until grade 3. In Singapore, however, multiplication and division are specifically taught beginning in grade 1, and in Finland, these operations are explicitly taught during the 1–2 grade span.

FINLAND

Numbers and Calculations Grades 1–2

- multiplication and multiplication tables
- division, using concrete tools

SINGAPORE

Multiplication and Division Primary 1

Include:

- multiplication as repeated addition (within 40),
- use of the multiplication symbol (×) to write a mathematical statement for a given situation,
- division of a quantity (not greater than 20) into equal sets:
  - given the number of objects in each set,
  - given the number of sets,
- solving 1-step word problems with pictorial representation.

Exclude:

- use of multiplication tables,
- use of the division symbol (÷).

While at first this appears to be a rigor issue, this is not the case. By the end of grade 6, U.S. students are expected to master all four operations with whole numbers, fractions, and decimals. By contrast, at grade 6, Singapore excludes certain sub-categories of division of fractions, and Finland excludes both multiplication and division of fractions and decimals for the grades 3–5 span.

When compared to Singapore and Finland, Indiana standards and U.S. standards such as the NCTM focal points, progress more slowly. They deal with operations in grades 1–3, then move more quickly in grades 4–6. This means that U.S. students have fewer years to build concepts and develop skills with the arithmetic operations. Their introduction to the operations of multiplication and division is delayed, yet by the end of grade 6, U.S. students are expected to exceed the level of work in the four operations that students in Singapore and Finland are accountable for.

While this is a typical timeline for the treatment of the four operations in the United States, it may be helpful to consider strategic adjustments at both ends of the grade span.

> **Suggestion:** Review the Singapore treatment of operations in grades 1–6 and consider adjusting the timing of this subject matter in the *Indiana Standards for Mathematics*.

Another difference between Singapore and Indiana is the level of specificity given in the standards for the four operations. Here are examples of the Singapore syllabus and the *Indiana Standards for Mathematics* for operations with fractions.

SINGAPORE

Concept of Fraction as Division Primary 5

Include:

- association of a fraction with division,
- conversion between fractions and decimals.

Four Operations Primary 5

Include:

- addition and subtraction of proper fractions **without using calculators,**
- addition and subtraction of mixed numbers,
- multiplication of a proper fractions and a proper/ improper fraction **without using calculators**,
- multiplication of an improper fraction and an improper fraction,
- multiplication of a mixed number and a whole number,
- division of a proper fraction by a whole number **without using calculators**,
- solving word problems involving the 4 operations.

Exclude:

- calculations involving more than 2 different denominators,
- multiplication of a mixed number by a proper fraction/improper fraction/mixed number,
- division of an improper fraction/mixed number by a whole number/ proper fraction,
- division by an improper fraction/mixed number
- (Denominators of given fractions should not exceed 12, for calculations without using calculators.)

INDIANA

Number Sense and Computation Grade 5

5.1.5 Add and subtract decimals and fractions, including fractions with different denominators and mixed number using a standard algorithmic approach.

Example:

$3\frac{4}{5} - 2\frac{2}{3} = ?$

5.1.6 Multiply fractions using a standard algorithmic approach.

Example:

Find $\frac{3}{4}$ of $\frac{2}{5}$. Explain when the product is smaller than the factors.

On the whole, Singapore has a larger number of more detailed operations standards than does Indiana, including which concepts should not be included. Indiana has fewer standards but they are shown with specific examples.

Suggestion: There is always a question as to the amount of specificity that is necessary, but consider adding operations standards where appropriate and making them more specific.

- *Difference* — **Focus on multi-step problems**

The Singapore curriculum emphasizes solving word problems, defining explicitly for teachers if the problems should be one-step or multi-step word problems. This progression is logically developed within their standards. Singapore's emphasis on multi-step word problems requires students to build and demonstrate a deeper understanding of the concepts.

SINGAPORE

Addition and Subtraction Primary 1

- solving 1-step word problems involving addition and subtraction within 20.

Multiplication and Division

- solving 1-step word problems with pictorial representation.

Addition and Subtraction Primary 2

- solving up to 2-step word problems involving addition and subtraction.

Multiplication and Division

- solving 1-step word problems involving multiplication and division within the multiplication tables.

(continues)

(continued)

Primary 3

Addition and Subtraction

- solving up to 2-step word problems involving addition and subtraction.

Multiplication and Division

- solving up to 2-step word problems involving the 4 operations.

Primary 4

Multiplication and Division: Whole Number

- solving up to 3-step word problems involving the 4 operations.

Multiplication and Division: Fractions

- solving up to 2-step word problems involving addition, subtraction and multiplication.

Multiplication and Division: Decimals

- solving up to 2-step word problems involving the 4 operations.

Primary 5

Percentage

- solving up to 2-step word problems involving percentage.

Ratio

- solving up to 2-step word problems involving ratio.

Volume of Cube and Cuboid

- solving up to 3-step word problems involving the volume of a cube/ cuboid.

Primary 6

Speed

- solving up to 3-step word problems involving speed and average speed.

Data

- solving 1-step problems using information presented in pie charts.

Finland does not have the same sequence of development as Singapore but does include objectives that infer students are solving problems.

FINLAND

Description of Good Performance at the End of Second Grade

- understand addition, subtraction, multiplication, and division and know how to apply them to everyday situations

Description of Good Performance at the End of Fifth Grade

- know how to depict real-world situations and phenomena mathematically by comparing, classifying, organizing, constructing, and modelling
- know how to present mathematical problems in a new form; they will be able to interpret a simple text, illustration, or event and to make a plan for solving the problem

Indiana includes problem solving as one of its process standards but does not emphasize the progression of problem solving in its standards in a similar manner to Singapore.

INDIANA

Algebra and Functions Grade 2

2.2.1 Write and solve single and multi-step open number sentences that represent addition and subtraction word problems.

Number Sense and Computation Grade 6

6.1.6 Solve problems involving addition, subtraction, multiplication and division of positive fractions and decimals and explain why a particular operation was used for a given situation.

Algebra and Functions Grade 6

6.2.1 Write and solve one-step linear equations and inequalities in one variable.

Suggestion: Review the development of word problems. Consider introducing problems that ensure students are exposed to problems involving more than one-step.

- *Difference* — **Development of the concept of quantity and relationships between quantities**

In Singapore classrooms, you will see teachers using both magnitudes without numbers (Which is longer?) and magnitudes with units and numbers. A quantity expresses a magnitude using units and the number of units (3 inches of length). This use of numbers is frequent in Singapore and coherently developed.

Singapore develops a coherent progression of the concept of quantity from:

- early grades in the area of measurement (money, distance, time, mass) to
- middle grades in the area of rates (speed, price, percent) and variables (as an extension of quantities from measurement) to
- high grades with functions and their graphs (to a scientist, the coordinates of a graph represent a measurable quantity, (cm., sec., gm.).

This progression offers the chance for greater efficiency by requiring less time in upper grades and algebra courses for the development of the concepts of variable and function.

Students study "quantities" and the relationships among quantities as the content of measurement. Thus, they study quantities measured by distance measures, time and money. Attention to units is given a critical focus, and this early work with units develops the idea of variables. For instance, in early elementary, Singapore students work with compound units (relationships between units) starting in grade 3.

SINGAPORE

Length, Mass and Volume — Primary 2

- use of the appropriate measures and their abbreviations cm, m, g, kg, *l*,
- solving word problems involving length/ mass/ volume.

Money

- converting an amount of money in decimal notation to cents only, and vice versa.

Length, Mass and Volume — Primary 3

- conversion of a measurement in compound units to the smaller unit, and vice versa,
- solving word problems involving length/ mass/ volume/ capacity.

Time

- conversion of time in hours and minutes to minutes only, and vice versa,
- finding the duration of a time interval,
- finding the starting time/ finishing time,
- solving word problems involving addition and subtraction of time given in hours and minutes.

Fluency with quantities, i.e. numbers with units, also supports learning fractions. The units denominate what is to be counted by the number and the number numerates how many. This is an important concept in studying fractions.

"Rate" and proportional relationships are built, in large part, on a foundation of work with quantities. In Singapore there is also more emphasis on the relationship between quantities. In this section on speed, notice the focus on the "relationship between distance, time, and speed" rather than merely solving problems to find one from the others.

SINGAPORE

Distance, Time and Speed Primary 6

Include:

- concepts of speed and average speed,
- relationship between distance, time and speed
  - Distance = Speed × Time
  - Speed = Distance ÷ Time
  - Time = Distance ÷ Speed
- calculation of speed, distance or time given the other two quantities,
- writing speed in different units such as km/h, m/min, m/s and cm/s,
- solving up to 3-step word problems involving speed and average speed.

Exclude:

- conversion of units, e.g. km/h to m/min.

From this work on the development of quantity, the conceptual foundation of functions is built. This is very much an engineering/science perspective on functions, rather than a formal mathematical conception. To an engineer, a variable is something you measure; it typically has units of measurement that define the units of the domain and the meaning of the numbers on the coordinates of a graph. Functions are formalized in electives in secondary, years 3–4 (roughly, at ages 15–16) for those who are specializing in STEM subjects.

In the United States, in general, students in earlier grades are not introduced to the concept of quantity and relationships between quantities. Discussing a quantity as a magnitude using units and the numbers of units is sporadic in the U.S. The *Indiana Standards for Mathematics* do not make explicit the incorporation of numbers with units into their progression of concepts in elementary grades.

Suggestion: Consider building this coherence from the early grades through minor revisions to the measurement standards and then extending it in the standards relating to rates, ratios, proportionality and linear functions. See the next item, "Treatment of rates, ratios and percents—development of proportional thinking," for more details on that section.

- *Difference* — **Treatment of rates, ratios and percents—development of proportional thinking**

The Singapore syllabus introduces rates, ratios and percents across four instructional years. Percents and ratio begins in grade 5 and is built on in grade 6. Problems of rate (focusing on speed) are also introduced in grade 6 with additional work on rate, ratio, and proportion in the next two years of secondary school.

SINGAPORE

Rates, Ratios, and Percentage Primary 5

- 6 bullet points on percent
- 7 bullet points on ratio

Rates, Ratios, and Percentage Primary 6

- 3 bullet points on percent
- 5 bullet points on ratio
- 5 bullet points on speed

Rates, Ratios, and Percentage Secondary 1

- 4 bullet points on ratio, rate, and proportion
- 6 bullet points on percentage
- 3 bullet points on speed
- 1 bullet point on functions and graphs

Rates, Ratios, and Percentage Secondary 2

- 2 bullet points on ratio, rate, and proportion
- 6 bullet points on percentage
- 3 bullet points on speed

With this more gradual accretion of concepts (and with the greater specificity about each concept within the standards), Singapore avoids the difficult task of working with three confusingly similar ideas all in one year. Also Singapore spends time on the relationship between quantities.

In grades 6 and 7, Indiana students work with proportional situations within the contexts of percents, ratios, and rates. Although percents, ratios, and rates overlap, there are some places where the concepts can be confused.

INDIANA

Number Sense and Computation Grade 6

6.1.7 Interpret ratios, model ratios, and use ratios to show the relative sizes of two quantities. Use the notations: a / b, a to b, and $a : b$.

Example:

A car moving at a constant speed travels 130 miles in 2 hours. Write the ratio of distance to time and use it to find how far the car will travel in 5 hours.

6.1.8 Recognize proportional relationships and solve problems involving proportional relationships.

Example:

Sam made 8 out of 24 free throws. Use a proportion to show how many free throws Sam would probably make out of 60 attempts.

6.1.9 Solve simple ratio and rate problems using multiplication and division, including problems involving discounts at sales, interest earned, and tips.

Example:

In a sale, everything is reduced by 20%. Find the sale price of a shirt whose pre-sale price was $30.

Note that standard 6.1.7 specifies ratio and 6.1.8 specifies proportion. Yet the two examples are conceptually and structurally alike. It may not be clear to the reader that these two standards are different in any interesting or important way.

The wording in standard 6.1.9 specifies ratio and rate, but the accompanying example is a percent problem.

In preparation for algebra, students need to understand that a proportional relationship is a relationship between two variables where one is a constant multiple of the other: let y and x be two variables and m be a constant. If $y = mx$, then y is proportional to x. Neither the Singapore *Mathematics Syllabus Primary* nor the *Indiana Standards for Mathematics* explicitly state that students should understand this.

Suggestion: In order for teachers and students to recognize the overlaps and distinctions among the proportionally-based concepts of percent, ratio, and rate, add a level of specificity across several grade levels.

- *Difference* — **Treatment of factors and multiples**

In the Singapore syllabus, there is explicit treatment of factors, common factors, multiples, and common multiples.

SINGAPORE

Whole Numbers: Multiplication and Division	Primary 2
▪ recognizing the relationship between multiplication and division	
Whole Numbers: Numbers up to 10,000	Primary 3
▪ odd and even numbers	
Fractions: Equivalent fractions	Primary 3
▪ writing the equivalent fraction of a fraction given the denominator or the numerator	
▪ expressing a fraction in its simplest form	
Whole Numbers: Factors and Multiples	Primary 4
▪ determining if a 1-digit number is a factor of a given number	
▪ listing all factors of a given number up to 100	
▪ finding the common factors of two given numbers	
▪ recognizing the relationship between factor and multiple	
▪ determining if a number is a multiple of a given 1-digit number	
▪ listing the first 12 multiples of a given 1-digit number	
▪ finding the common multiples of two given 1-digit numbers	

Students are expected to learn and use the concepts of factors and multiples in grade 4, one year following the grade 3 conceptual emphasis on multiplication and division of whole numbers. This work with factors and multiples forms the underpinnings of students' work in arithmetic operations with the traction form that figures largely in grades 4 and 5.

Below are the *Indiana Standards for Mathematics* that address factors and multiples.

INDIANA

Number Sense and Computation — Grade 2

2.1.5 Identify odd and even numbers and determine whether a set of objects has and odd or even number of elements.

Number Sense and Computation — Grade 4

4.1.2 Compare and order fractions by using the symbols for less than (<), equals (=), and greater then (>).

Number Sense and Computation — Grade 5

5.1.3 Identify prime and composite numbers.

5.1.5 Add and subtract…fractions with different denominators and mixed numbers using a standard algorithmic approach.

The *Indiana Standards for Mathematics* do not explicitly address divisibility, common factors, and common multiples, which are essential to calculating with fractions. Students compare and order fractions in grade 4 (where, although the example given involves simple fractions, the denominators are different). Then go on to add and subtract fractions with different denominators in grade 5, without any explicit mention of factors and multiples in the standards.

Suggestion: Consider explicitly stating in the standards, probably in grade 4, what students are expected to understand and be able to do with factors, multiples, common factors, and common multiples.

- *Difference* — **Treatment of rounding and estimating**

Finland's National Core Curriculum lists "evaluating, checking, and rounding the results of calculations" in the core contents for grades 3–5. At grades 6–9, the core contents include "rounding and estimation; using a calculator."

The Singapore syllabus is explicit in various places about estimation and approximation skills. Rounding appears in grade 4, where skills include "rounding off [whole] numbers to the nearest 10 or 100" and "rounding off [decimal multiplication and division] answers to a specified degree of accuracy."

While Indiana students are required to perform tasks that support the learning and use of rounding skills (count by hundreds, plot numbers on a number line, recognize real-world measurements as approximations, etc.), the standards never explicitly require the students to round.

> **Suggestion**: Consider specifying some rounding skills in the *Indiana Mathematics Standards*, beginning at grade 4. Use examples that show that "rounding" is a building block for understanding approximation and interpreting decimal results within our base-ten system and also reinforces other tools for approximation. Also include more emphasis on checking and estimating computations.

- *Difference* — **Treatment of decimals in grade 4**

The Singapore grade 4 standards are explicit in three places about decimals as an embodiment of place value and working with decimals, which provide teachers with clarity on what students should be able to do.

SINGAPORE

Decimals up to 3 Decimal Places Primary 4

Include:

- Notation and place values (tenths, hundredths, thousandths)
- Identifying the values of the digits in a decimal
- Use of the number line to display decimals
- Comparing and ordering decimals
- Conversion of a decimal to a fraction
- Conversion of a fraction whose denominator is a factor of 10 or 100 to a decimal
- Rounding off decimals to the nearest [whole, tenth, hundredth]

Indiana standard 4.1.3 addresses working with decimals, however, the standard itself puts no particular focus on place value.

INDIANA

Number Sense and Computation Grade 4

4.1.3 Interpret and model decimals as parts of a whole, parts of a group, and points and distances on a number line. Write decimals as fractions.

If students are modeling and interpreting decimal numbers, this work should include the understanding and use of the "10 to 1" relationship between each pair of adjacent places.

Also, in grade 5, Indiana students count, read, write compare, and plot decimals to 3 decimal places (5.1.1), compare and order decimals, fractions and percents (5.1.2), and add and subtract decimals (5.1.5). This work builds on the grade 4 introduction to decimals, which is contained in a single standard.

Suggestion: Consider writing additional standards about decimals and tie them to place value explicitly.

- *Difference* — **Timing of odd and even numbers**

At the end of grade 2, Finnish students are expected to "know about odd and even numbers." They are also expected to work with "multiplication and the multiplication tables," as well as "division, using concrete tools." In Singapore, odd and even numbers are treated in grade 3, with work in multiplication and division beginning in grade 1.

Indiana Standards for Mathematics place the understanding of odd and even numbers with skip counting patterns in grade 2:

INDIANA

Number Sense and Computation Grade 2

2. 1. 5 Identify odd and even numbers and determine whether a set of objects has an odd or even number of elements.

2. 1. 2 Count by ones, twos, fives, tens, and hundreds to at least 1,000.

Thus, students are introduced to odd and even numbers in grade 2, a year before multiplication and division are introduced. To understand the underpinnings of the concept of odd and even, rather than simply naming a number as odd or even, students will need the concept of multiplication—"even numbers are multiples of two"—and division—"odd numbers make groups of two with a remainder."

Suggestion: Consider putting work with odd and even numbers in the same instructional year that students develop the concept of multiplication and division.

- *Additional considerations*

Specificity of Standards

In some cases, the *Indiana Standards for Mathematics* specify the number ranges for the specific standards (e.g., multiplication facts to 10) and in others these are not included.

Consider reviewing the standards with an eye to making them more specific about the ranges of numbers that apply.

More Examples for Complex Standards

Some Indiana standards summarize a list of activities and subcategories related to a specific concept, such as 4.1.1 which deals with counting, reading, writing, comparing and plotting whole numbers using words, models, number lines and expanded form:

Consider providing two or three examples (rather than just one) to better illustrate the range of student activities and mathematical representations addressed in the standard.

Revise Example in Standard 4.1.5

Indiana standard 4.1.5 states that students will "multiply numbers up to at least 100 by a single-digit number and by 10 using a standard algorithmic approach." The example that is paired with this standard shows a multiplication problem with two 2-digit factors: 86×54. (Note that neither factor is 10.)

Consider changing the example to include a factor of 10.

Include Grade 2 Standard for Comparing and Ordering Numbers

Students at grade 1 are required to name numbers one more or one less than a given number to 100 (Standard 1.1.3). Students at grade 3 are asked to compare and order fractions (Standard 3.1.4). At grade 2, although students are asked to use place value to show numbers 10 more or 10 less than a given number, 10 to 90 (Standard 2.1.3), the standard is in the place value category. This creates a grade level gap in the comparing and ordering numbers subcategory.

Consider relocating Standard 2.1.3 to the comparing and ordering numbers sub-category. It is very similar to Standard 1.1.3. Then write a new standard for the place value sub-category to avoid a gap at grade 2: "Recognize numbers to at least 1000 as groups of hundreds, tens, and ones."

Standard 2: **Algebra and Functions**

- *Assets*

The *Indiana Standards for Mathematics* that pertain to number patterns, grades K–4, show a nice incremental progression. Starting with number and shape patterns in kindergarten, the standards introduce a new arithmetic operation each year, ending with division at grade 4. These parallel nicely the conceptual focus on pattern and operations seen in each year through grades 1–4.

Indiana standard 7.2.7 states, "Identify situations that involve proportional relationships, draw graphs representing these situations and recognize that these situations are described by a linear function in the form $y = mx$ where the unit rate m is the slope of the line." This standard does a nice job of explicitly connecting proportional relationships (studied in some detail in grade 6) to linear functions.

Similarly, standard 8.2.6 encourages the teaching and learning of multiple representations of functions: "Translate among tables, equations, verbal expressions and graphs of linear functions."

Both these standards signal to the teacher that students are expected to make connections among topics and representations. The wording of these standards provides some safeguard against the tendency to teach the elements of function as discrete, unconnected ideas.

- *Difference* — **Treatment of rules of arithmetic**

Rules of arithmetic and number sentences are areas which are invisible in the Singapore or Finland standards, but their use classrooms has proven effective. Number properties and the properties of equality are major organizing ideas that are the foundation of the language of algebra used in arithmetic. The way these properties work together to define number and operations forms the foundation for algebra. This deserves a coherent progression which can lead to efficiencies in upper elementary, middle school and algebra.

Indiana includes the number properties in their standards. However the approach to these are from an operations with whole numbers point of view. The number properties especially the distributive property can help students understanding operations with negative numbers and are essential in solving equations and simplifying expressions. Having students justify each step in solving an equation using the number properties will help them understand how to keep the equation balanced.

Number sentences (equations) should be a routine focus of arithmetic. Each new topic studied should include understanding the associated number equations. The number properties and the properties of equality should be applied explicitly to each new topic in arithmetic. Young students should be asked to produce number sentences nearly as often as they are asked to produce calculated answers. Currently, this practice is not customary in U.S. classrooms.

Suggestion: Consider following the use of the rules of arithmetic beyond mental math activities. Consider having more emphasis on producing the equation, not the answer.

- *Difference* — **Early work with equations**

The *Indiana Standards for Mathematics* introduce algebra work in grades 1 and 2. This is done through the use of elements called "open number sentences" and "single- and multi-step open number sentences" (which are simple equations). From grades 1–4, Indiana students progress through the four arithmetic operations with writing and solving simple equations. Solving these equations consists of undoing an operation by knowing and using its inverse. In *Indiana Standards for Mathematics*, much of the conceptualization of pairs of inverse operations lives in Algebra and Functions.

Although it is not called algebra in Singapore, the fundamentals are there. There is an explicit focus on inverse operations in grade 1 for addition and subtraction and in grade 2 for multiplication and division.

So Singapore primary students do and undo operations under the headings of inverse operations and two-step problems, while Indiana children do it under the heading of algebra.

Suggestion: Consider renaming open number sentences and single and multi-step open number sentences to "simple equations" in the primary grades and using examples to show how simple they are.

- *Difference* — **Treatment of number patterns**

The *Indiana Standards for Mathematics* that pertain to number patterns show a solid progression in grades K–4. However, they do not explicitly extend into the upper grades to tie together number patterns with algebraic concepts.

INDIANA

Number Sense and Computation — Kindergarten and Grade 1

K.2.2 Create, extend, and give the rule for simple patterns with numbers and shapes.

1.2.2 Create, extend and give a rule for number patterns using addition.

Algebra and Functions — Grades 2–4

2.2.2 Create, extend and give a rule for number patterns using addition and subtraction.

3.2.2 Create, extend and give a rule for number patterns using multiplication.

4.2.2 Create, extend and give a rule for number patterns using multiplication and division

The Singapore standards also attend to number patterns, Grades K–4, but without any specificity about the sorts of patterns students should encounter in each grade.

SINGAPORE

Whole Numbers to 100 Primary 1

- number patterns

Whole Numbers to 1000 Primary 2

- number patterns

Whole Numbers to 10 000 Primary 3

- number patterns

Whole Numbers to 100 000 Primary 4

- number patterns

The Finnish standards specify only simple number sequences in grades 1 and 2 and interpretation and writing of number sequences in grades 3 to 5.

FINLAND

Algebra Grades 1–2

- simple number sequences

Good Performance at the End of Grade 2

- master the breaking down and assembly of numbers, comparison, and the formation of sums and number sequences;

Algebra Grades 3–5

- interpretation and writing of number sequences

Good Performance at the End of Grade 5

- know how to formulate and continue number sequences and to present correlations.

Both Singapore and Finland do, however, add to the demand regarding number patterns in later grades, connecting early learning in number patterns to later algebraic concepts.

SINGAPORE

Algebraic Representation and Formulae Secondary 1

- Recognizing and representing number patterns (including finding an algebraic expression for the nth term)

FINLAND

Algebra Grades 6–9

- study and formulation of number sequences

Functions

- know how to continue a number sequence according to the rule given and be able to describe the general rule for a given number sequence verbally

Suggestion: Consider extending work with patterns into upper grades.

- *Additional considerations*

Revise Example in Standards 2.2.3 and 3.2.3

Indiana's grade 2 and 3 examples for standards 2.2.3 and 3.2.3 (the commutative and associative properties) are more appropriate as illustrations of mental calculation. Currently, they read as follows:

> Example: Mentally add the numbers 5, 17, and 13, in this order. Now add them in the order 17, 13, and 5. Tell which order was easier and why.
>
> Example: Multiply the numbers 7, 2, and 5, in this order. Now multiply them in the order 2, 5, and 7. Tell which was easier and why.

Consider changing these examples so they are less focused on mental calculation and more about the arithmetic property of commutativity by rewording them as follows:

> Example: Add the numbers 5, 17, and 13 in this order. Now add them in the order 17, 13, and 5. Show that the results are the same. Explain why.
>
> Example: Multiply the numbers 7, 2, and 5, in this order. Now multiply them in the order 2, 5, and 7. Show that the results are the same. Explain why.

Revise Example in Standard K.2.1

The *Number Sense and Computation* standards and accompanying examples for Kindergarten specify that student work with addition and subtraction is consistently concrete: "Model addition by joining sets of objects," "model subtraction by removing objects," states K.1.5. In standard K.1.3 students play a one more/one less game by using actual dominoes.

In standard K.2.1, the example asks students to describe an addition relationship in a number sentence: 3 + 1 = 4. It is not clear whether "describe" means in strictly verbal terms—"Three and one more make four," although this would be consistent with the other standards and examples.

Assuming that standard K.2.1 means verbally and with concrete objects, consider adding "verbally" to the standard and specifying the use of "objects," rather than reading a hundreds chart, as in the example.

Revise Example in Standard 6.2.1

The present wording of the 6.2.1 example is:

> Example: The area of a rectangle is 143 cm^2 and the length is 11 cm. Write an equation to find the width of the rectangle and use it to solve the problem. Describe how you will check to be sure that your answer is correct.

Consider changing it to:

> Example: The area of a rectangle is 143 cm^2 and the length is 13 cm. Write and solve an equation for the width of the rectangle. Describe how you will check to be sure that your answer is correct.

This uses more standard "write and solve" wording and yields a rectangle with a length greater than its width.

Revise Example in Standard 8.2.3

The present wording of the 8.2.3 example is:

> Example: Use a scientific calculator to find the value of $3(2x + 5)^2$.

Consider changing it to:

> Example: Use a scientific calculator to expand $3(2x + 5)^2$.

Also, consider refraining from mentioning the use of a calculator, since 8.2.3 specifically asks students to be able to "simplify algebraic expressions involving powers."

Standard 3: **Geometry and Measurement**

- *Assets*

The *Indiana Standards for Mathematics* cover important concepts of transformations, symmetry, angles, lines, area, perimeter, surface area and volume. Mathematical tools and technology are suggested to enable student understanding of some of the concepts.

- *Difference* — **Emphasis on geometry and measurement**

Singapore deals with geometry and measurement as separate topics. Measurement leads to the development of quantity. The content covered in the Singapore syllabus gives more detailed development of the concepts with more connections to number and algebra.

Suggestion: Indiana specified a goal of depth over breadth. Thus decisions on when to add more detail to the standards are difficult ones, but consider reviewing the geometry and measurement sections of the standards for depth, especially concerning the measurement area and its relationship to developing the concept of quantity.

- *Difference* — **Treatment of transformations and constructions**

The Singapore standards have an emphasis on symmetry and transformations throughout the grades. The Singapore standards read as though they are laying the groundwork for later work with transformations in grades K–6.

SINGAPORE

Geometry: Patterns Primary 2

Include:

- making/completing patterns with 2-D cut-outs according to one or two of the following attributes
  - shape
  - size
  - orientation
  - colour

Geometry: Patterns Primary 3

Include:

- conversion of units, e.g. km/h to m/min.

Geometry: Symmetry Primary 4

Include:

- identifying symmetric figures,
- determining whether a straight line is a line of symmetry of a symmetric figure,
- completing a symmetric figure with respect to a given horizontal/vertical line of symmetry,
- designing and making patterns.

Exclude:

- finding the number of lines of symmetry of a symmetric figure,
- rotational symmetry.

Geometry: Tessellation Primary 4

Include:

- recognising shapes that can tessellate,
- identifying the unit shape in a tessellation,
- making different tessellations with a given shape,
- drawing a tessellation on dot paper,
- designing and making patterns.

The Finnish standards indicate there is regular, incremental work through the grades on concepts of transformation.

FINLAND

Geometry — Grades 1–2

- simple reflections and dilations

Geometry — Grades 3–5

- dilations and reductions; similarity and scale
- reflections across a line and around a point, symmetry, congruence, using concrete means

Geometry — Grades 6–9

- similarity and congruence
- geometric construction
- depictions of congruence: reflections, rotation, and transformation

The *Indiana Standards for Mathematics* also include work on transformations and constructions.

INDIANA

Geometry and Measurement — Grade 2

2.3.2 Identify and draw congruent two-dimensional shapes in any position.

Geometry and Measurement — Grade 3

3.3.2 Identify and draw lines of symmetry in geometric shapes and recognize symmetrical shapes in the environment.

Geometry and Measurement — Grade 7

7.3.1 Identify and use the following transformations: translations, rotations and reflections to solve problems.

Geometry and Measurement — Grade 8

8.3.2 Perform basic compass and straight edge constructions: angle and segment bisectors, copies of segments and angles, and perpendicular segments. Describe and justify the constructions.

The topics of transformations and symmetry might be introduced over a series of years.

> **Suggestion**: Symmetry has an important place in the coordinate plane. Finland has students working with dilations well in advance of high school. Consider building the idea of symmetry and transformation across grade levels.

- *Difference* — **Treatment of money concepts**

Singapore uses money to develop the concept of equivalence and operations. The standards detail the incremental conceptual work across the primary grades, including the concepts (equivalence and operations) and skills (notation) that form the basis of calculation with money:

SINGAPORE

Measurement: Money Primary 1

Include:

- Identifying coins and notes of different denomination
- Matching a coin/note of one denomination to an equivalent set of coins/notes of another denomination
- Telling the amount of money
  - in cents up to $1.00
  - in dollars up to $100
- Use of the symbols $ and ¢
- Solving of word problems involving addition and subtraction of money in dollars only (or in cents only).

Measurement: Money Primary 2

Include:

- Counting the amount of money in a given set of notes and coins
- Reading and writing money in decimal notation
- Comparing two or three amounts of money
- Converting an amount of money in decimal notation to cents only, and vice versa
- Solving word problems involving money in dollars only (or in cents only)

Measurement: Money Primary 3

- Addition and subtraction of money in decimal notation
- Solving word problems involving addition and subtraction of money in decimal notation

The *Indiana Standards for Mathematics* on money include the following:

INDIANA

Geometry and Measurement Grade 1

1.3.3 Give the value of a collection of pennies, nickels, and dimes up to $1.00.

Geometry and Measurement Grade 2

2.3.5 Find the value of a collection of pennies, nickels, dimes, half-dollars, and dollars.

While the examples for standards 3.2.1 and 6.2.2. (grades 3 and 6, respectively) involve money, the two standards listed on the previous page for grades 1 and 2 are the only actual standards that focus on work with money.

Suggestion: Consider extending work with money across grades 1–3 and incorporating specific, grade level requirements about equivalence, operations with money (including word problems) and use of decimal notation and the ¢ sign.

- *Difference* — **Treatment of grade 2 geometry**

The Indiana grade 2 standards in geometry emphasize the vocabulary of geometry.

INDIANA

Geometry and Measurement Grade 2

2.3.1 Recognize, identify, and describe attributes of common shapes and solids (e.g., the same size and type of shape; number of sides, edges and vertices; location).

2.3.2 Identify and draw congruent two-dimensional shapes in any position.

The Singapore geometry standards for grade 2 focus less on vocabulary and instead emphasize concrete activities.

SINGAPORE

Geometry: 2-D and 3-D Figures Grade 2

Include:

- Identifying, naming and describing
  - semicircle
  - quarter circle
- Identifying the basic shapes that make up a given figure
- Forming different 2-D figures with cutouts of
  - rectangle
  - square
  - triangle
  - semicircle
  - quarter circle
- Forming different 3-D figures with concrete models of
  - cube
  - cuboid
  - cone
  - cylinder only, and vice versa
- Copying figures on dot grid or square grid.

Suggestion: Consider replacing standard 2.3.1 with a pair of grade 2 geometry standards (one for 2-D and one for 3-D shapes) that focus more on concrete activities.

- *Difference* — **Timing of symmetry**

Indiana students work with symmetry before they study intersecting lines and angles.

INDIANA

Geometry and Measurement Grade 3

3.3.2 Identify and draw lines of symmetry in geometric shapes and recognize symmetrical shapes in the environment.

Geometry and Measurement Grade 4

4.3.1 Identify, describe, and draw parallel and perpendicular lines.

4.3.2 Identify, describe, and draw right angles, acute angles, obtuse angles, straight angles, and rays using appropriate tools and technology.

The Singapore standards explicitly introduce angles and line relationships in grade 3, and begin work with symmetry in grade 4.

SINGAPORE

Geometry: Perpendicular and Parallel Lines Grade 3

Include:

- Identifying and naming perpendicular and parallel lines
- Drawing perpendicular and parallel lines on square grids

Geometry: Angles Grade 3

- Angle as an amount of turning
- Identifying angles in 2-D and 3-D objects
- Identifying angles in 2-D figures
- Identifying right angles, angles greater than/ smaller than a right angle

Geometry: Symmetry Grade 4

- Identifying symmetric figures
- Determining whether a straight line is a line of symmetry of a symmetric figure
- Completing a symmetric figure with respect to a given horizontal/vertical line of symmetry
- Designing and making patterns

Suggestion: Consider introducing angles, and parallel and perpendicular lines before addressing symmetry.

- *Difference* — **Timing of angles**

The *Indiana Standards for Mathematics* introduce angles in grade 4 (prior to that, K–2 students classify and describe plane and solid figures in terms of their number of vertices).

INDIANA

Geometry and Measurement — Grade 4

4.3.2 Identify, describe, and draw right angles, acute angles, obtuse angles, straight angles, and rays using appropriate tools and technology.

Singapore introduces angles more gradually, starting in grade 3.

SINGAPORE

Geometry: Angles — Grade 3

- Angle as an amount of turning
- Identifying angles in 2-D and 3-D objects
- Identifying angles in 2-D figures
- Identifying right angles, angles greater than/ smaller than a right angle

Geometry: Angles — Grade 4

- Using notation such as $\angle ABC$ and $\angle x$ to name angles
- Estimation and measurement of angles in degrees
- Drawing an angle using a protractor
- Designing and making patterns
- Associating

  $\frac{1}{4}$ turn/right angle with 90°

  $\frac{1}{2}$ turn with 180°

  $\frac{3}{4}$ turn with 270°

  a complete turn with 360°.

Suggestion: Consider introducing angles earlier and building conceptual knowledge more gradually.

- *Difference* — **Timing of units work**

Indiana grade 2 students begin work with units.

INDIANA

Geometry and Measurement Grade 2

2.3.3 Estimate and measure length to the nearest inch, foot, yard, centimeter, and meter, selecting appropriate units for the given situation, and use the relationships within the units to express answers in different units.

2.3.4 Describe relationships of time (seconds in a minute, minutes in an hour, hours in a day, days in a week, and days in a year) and tell time to five minute intervals.

The relationships mentioned in standard 2.3.4 are based on multiplication and division—operations which are fully introduced in grade 3.

Singapore and Finland start the development of multiplication in grade 1 and emphasize units and quantities in early elementary.

Suggestion: Move the treatment of units to follow the introduction of multiplication and division. Also consider the suggestion about the placement of multiplication and division in grade 1 and the development of the concept of quantity, as this will effect the measurement section of *Indiana Standards for Mathematics* in primary grades.

- *Additional considerations*

Repeat Work on Surface Area and Volume

Indiana students do a lot of work with perimeter, area, surface area, and volume in grades 4–8. There are some standards at grade 7 that appear to be restatements of requirements from previous years. Below are the two grade 7 measurement standards. Where concepts or skills are restatements of standards from earlier grades, they are shown in italics.

7.3.5 Develop and use formulas for finding the perimeter and area of basic two-dimensional shapes (*rectangles* (4.3.5), *parallelograms* (5.3.5), *trapezoids* (5.3.5), *triangles* (5.3.5), *circles* (6.3.6)) and the surface area and volume of basic three-dimensional shapes (*prisms* (5.3.6) and *cylinders* (6.3.6)).

7.3.7 Estimate and compute the area of *more complex or irregular two-dimensional shapes by dividing them into more basic shapes* (5.3.5).

Consider moving the work with the surface area and volume of cylinders to grade 7. In this manner, grade 7 students would be required to continue working with circles, while adding important new material.

Revise Standard 4.3.1.

Standard 4.3.1 states that students will "identify, describe, and draw parallel, perpendicular, and intersecting lines using appropriate mathematical tools and technology." This wording is not explicit about perpendicular lines being a special case of intersecting lines.

Consider rewording the standard as follows: "Identify, describe, and draw pairs of parallel, perpendicular, and non-perpendicular intersecting lines using appropriate mathematical tools and technology."

Numbering in Grade 7 Measurement Standard

The grade 7 standard about similarity relationships appears twice, designated as both 7.3.3 and 7.3.4.

There is a gap between standard 7.3.5 and 7.3.7 (no 7.3.6 standard appears).

Standard 4: **Data Analysis and Probability**

- *Assets*

Indiana does a good job of development of the topic of data starting from collecting and representing data using pictures in grade 1 to analyzing, interpreting and displaying data in box-and-whisker plots in grade 8. The concepts in the data section in most cases are correlated to the numbers and operations studied in the Number Sense and Computation section.

- *Difference* — **Treatment of probability**

Indiana students are required to meet these standards at grades 6, 7, and 8.

INDIANA

Data and Probability Grades 6–8

6.4.3 Recognize and represent probabilities as ratios, measure of relative frequency, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable.

7.4.5 Use theoretical probability and proportions to make approximate predictions.

8.4.6 Describe and apply the addition rule for probabilities for simple events that are mutually exclusive and for simple events that are not.

8.4.7 Compute probabilities of events from simple experiments with equally probable outcomes, using such methods as organized list, tree diagrams, and area models intervals.

In contrast, Finland treats probability in grades 6–9. By the end of grade 8, students "determine the number of possible events and organize a simple empirical investigation of probability; understand the meaning of probability and randomness in day-to-day situations."

FINLAND

Data processing, Statistics, and Probability — Grades 3–5

- know how to clarify the number of different events and alternatives, and to judge which is an impossible or certain event.

Probability and Statistics — Grades 6–9

- concept of probability
- frequency and relative frequency
- determining die average, mode, and median
- concept of dispersion
- interpretation of diagrams
- gathering and adapting information, and presenting it in a usable form

Singapore does not introduce probability until grade 8. In addition, the probability of mutually exclusive events is taught in grade 9, a year later than in Indiana.

SINGAPORE

Probability — Secondary 1

- probability as a measure of chance
- probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability)

Suggestion: Consider adding a standard similar to Finland's or Singapore's, which highlights probability as, first and foremost, a measure of chance—and the importance of understanding the concept of probability.

- *Difference* — **Timing of data displays**

Stem-and-leaf plots allow estimation (or calculation) of measures of central tendency and make it easy to observe outliers. Indiana students are introduced to stem plots in grade 4, but do not take up measures of central tendency until grade 6.

INDIANA

Data and Probability Grade 4

4.1.7 Construct and analyze line plots and stem-and-leaf plots,

Data and Probability Grade 6

6.4.2 Compare the mean, median and mode for a set of data and explain which measure is most appropriate in a given context.

Singapore introduces students to the stem-and-leaf plot in grade 8—the same year students learn to calculate mean.

SINGAPORE

Data Analysis **Secondary 1**

Include:

- interpretation and analysis of:
  - dot diagrams
  - stem-and-leaf diagrams
- mean, mode and median as averages
- purposes and use of mean, mode and median
- calculation of the mean for grouped data

Suggestion: Consider introducing stem-and-leaf plots after students have been introduced to measures of central tendency (mean, median, mode, and range).

- *Additional considerations*

Sequence dealing with circle graphs

Circle graphs are first mentioned at grade 6 in standard 6.4.1 which suggests students have already learned to use circle graphs: "Choose the appropriate display for a single-variable set of data from bar graphs, line graphs, circle graphs, and stem-and-leaf plots. Justify your choice."

Consider adding a standard in grade 6 about constructing and analyzing circle graphs.

Revise Standard 2.1.7

This standard states "Compare data displayed in tables and picture graphs within the table or graph and to other tables and graphs." This wording specifies making comparisons both within and across displays.

Consider a rewording the standard to the following "Make comparative statements about the categories in a single data set/display; make comparative statements about two data sets/displays that address a single question." This should be followed by two examples—one for each subpart of the standard.

Review Example in Standard 7.4.4

This standard includes the requirement that students be able to analyze the ways in which data displays may be misleading. Below is the accompanying example.

> **Example:**
>
> A company displays a bar graph of the company's sales and indicates that sales have more than doubled since last year. Upon analyzing the graph, you notice that sales have in fact increased from $5.5 million to $6.2 million. Explain how the company used the graph to show that sales doubled.

The example would be more helpful if it included a bar graph with an abbreviated vertical axis (dollars in sales). That axis might show values starting at $5 million—thus the bar representing $6.2 million will be a bit more than twice the height of the bar representing $5.5 million, leading to the false claim that sales have doubled. Without including the graph and allowing it to show *the manner in which* it is misleading, the example is difficult to understand.

Consider completing the example by providing the graph or choosing another example that does not require the inclusion of a graph to make its case.

High School

The Indiana mathematics standards for high school courses cannot be correlated directly to Singapore's *Secondary Mathematics Syllabuses* or Finland's *National Core Curriculum for Upper Secondary* because of overall structural differences in the schools' education systems and the organization of course sequences.

Because of these structural differences, this part of the report is structured around three areas.

- Comparison of the Secondary Education Systems of the U.S., Singapore, and Finland
- Critical Differences
- Comparison of the Mathematics

Comparison of Secondary Education Systems

The U.S. high school system, as reflected in Indiana's achievement standards, differs from international systems (Asian and European) in important ways.

- ### Multi-year programs versus courses

  Both Singapore and Finland employ a typical Asian/European approach in that separate programs are well-defined across multiple years of study and build toward a specific outcome. Defined by detailed syllabi, each program includes a specific sequence of work and examinations. The result of completing a program does more than simply qualify students for employment; rather, the programs culminate in credentials that lead to specific careers.

  Within each program, courses (though distinct units of study) are an integral part of the whole. Individual courses, as we think of them in the United States, do not exist as separate entities outside the context of the complete program.

  In the U.S., the route of study in high school course is only determined by a generic set of college eligibility rules, or requirements. The U.S. systems, including Indiana's, specify standards for numerous courses, rather than comprehensive programs, with only a loose sequencing within and across courses. The actual sequence and combination of courses is open to chance and buried in high school master schedules, counseling sessions, and college eligibility and placement policies. For most U.S. students, the path through mathematics is unclear. Many students do not know when they need to choose a direction, much less what direction their choice will permit. There are few programs that move students through the upper grades to career-readiness in any well-organized way.

- ## Age comparisons

Finland, Singapore, and most international countries organize students' education around specialized study that begins shortly after age 16 and continues through age 19. Finland's *National Core Curriculum for Basic Education* defines course work through age 16, after which students begin Upper Secondary work. Singapore's *Mathematics Syllabus Primary* defines course work through age 12 and the *Secondary Mathematics Syllabuses* through age 16, after which students begin Pre-University work.

Whether it is considered Upper Secondary or Pre-University, it is typical for students to engage in approximately three years of work at this level in both Finland and Singapore. Thus, by age 19, the education level of these students is roughly equivalent to that of incoming college sophomores in the United States.

Because of the above differences, any results comparing U.S. high school graduates to those of Finland or Singapore (who have received an additional year of schooling) would be misleading. Hence, the Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) compare students of the same age rather than grade level.

- ## Different programs within the systems

By (or near the end of) age 16, students in Finland and Singapore have begun course work in specialized academic programs. These programs are carefully and specifically organized by career orientation, including preparing students for university study.

After completing the *Secondary Mathematics Syllabuses*, about a fourth of Singapore students enter programs considered Pre-University. Within the Pre-University program, some students will follow more intensive mathematics pathways (H2 and H3) to pursue majors in science, technology, engineering, and mathematics. Others may engage in a less intensive sequence (H1) to prepare for careers in accounting and business management, social sciences, arts and letters. Singapore includes serious statistics for all students, but reduces the calculus preparation content for H1 students. The examinations for H2 and H3 are considerably different from the examinations for H1. For those Singapore students who do not enter Pre-University, Polytechnics and Technical Education are available.

In Finland, 92% of students who complete Basic Education at age 16, continue directly to General Upper Secondary or Vocational Upper Secondary course work. Upper Secondary consists of compulsory, specialization, and applied courses. Within the General Upper Secondary, the specialization courses allow for more rigorous study.

In the U.S., college-bound students take Algebra I, Geometry, and Algebra II to meet minimum entrance requirements. Many are guided further through an Algebra II course specifically aimed toward a Calculus sequence that is suitable for STEM majors.

- **Mobility within the systems**

In Singapore, diagonal pathways allow students to cross over from one academic program to another. These pathways are not remedial; rather they teach content included in the destination program that was missing from the originating program. The content of less academic programs is sufficiently aligned to make such laterally upward movement efficient for motivated students. Crossing over, however, does add to the overall time it takes to reach the university level.

In Finland, the Upper Secondary program includes a vocational or "trades" path, in addition to its General Upper Secondary program. While transitions can be made between the basic and advanced courses in Finland, crossing from one program to another is unusual and more difficult.

The structure of the mathematics programs in the U.S. allows high school students to fall short of meeting the necessary requirements for college eligibility. Many students transition out of the university track by default through either failing or not taking the necessary courses. The U.S. also provides no similar option for secondary students who have a special interest in vocational training.

- **Interdisciplinary courses**

Singapore strives to reinforce the natural math-science association; math courses are interconnected with science courses. Furthermore, all students are required to take the Design & Technology course. This project-based course requires students to apply knowledge of science and mathematics to complex problems, within a framework of a well defined design process. The syllabus of the Design & Technology course has been included as an appendix to illustrate the content in that course. Even though it is not a specific part of the mathematics or science syllabi, this compulsory course is a component of each student's education.

Finland's secondary system has a similar applied component. These applied courses may be interdisciplinary courses, methodological courses, vocational courses, or other studies organized by local educational agencies.

In the United States, the interrelationship of subjects is not emphasized. Mathematics and science courses are not allied strongly; hence each must stand on its own.

- **Alignment of curriculum and assessment**

In Singapore, secondary programs are defined by syllabi that also double as examination syllabi. They are specifically designed to align. Therefore, programs are constrained by the time available for examination. Singapore programs focus on learning, practicing, and studying precisely what will be examined, as do Advanced Placement (AP) courses in the U.S.

These syllabi are written to answer a student's question, "What should I study to do well on the examination?" The exams reward studying, because students know how and what to study. Students can achieve higher levels of performance by studying wisely—using their time, the syllabi, and prior exams as a guide.

The majority of U.S. courses do not have such alignment between course objectives and examination goals. The typical state test will always underestimate the level of achievement students might otherwise have shown on an examination in a transparent system with specific syllabi and study guidelines.

In the U.S., assessments are designed to distribute test takers on a scale rather than to examine their achievement (as in AP exams). Test items come from a domain sampling approach that uses frameworks instead of an exam syllabus. As a consequence, students face a black box and are unable to prepare specifically for their assessments. In fact, they have very little control over their performance other than whatever they have done to develop the concepts being measured.

Critical Differences

The following is a discussion of some of the critical differences in the three secondary education systems, along with suggestions for integrating some of the strengths of the Singapore and Finland systems into the Indiana system.

- *Difference* — **Timing of "forks in the road"**

  In Singapore, the fork in the road happens around age 16, when 75% of students enter one of three programs which could result in entering the university. The remaining 25% specialize in technical programs, which lead to other certifications, but can also lead indirectly to university, for students who wish to make the transition. For the 25% in the technical program, further mathematics is embedded in technical coursework that is very hands-on and applied. The highest academic mathematics completed in the technical program is the equivalent of U.S. grades 9 and 10.

  In Finland, there is also a fork at the age 16. At that time students are done with the 9 years of Basic Education and enter either General Upper Secondary or Technical Education.

  Up until recently, U.S. High School graduation or "leaving" requirements have been de facto less than the Singapore requirements for students in the technical program (U.S. grades 9 and 10). The fork in the U.S. road happened around age 16 or 17, when students going to college enrolled in Algebra II.

  More recently however, U.S. policy trends are pushing for all students to go well past Algebra I and Geometry into topics historically associated with Algebra II, so that all students can enter college without remediation. If this is to become the default high school graduation requirement, then Algebra II will become the highest common course for all students. Based on the content of the highest common courses in Singapore and Finland, Algebra II may be far too demanding for this purpose. Since Algebra II has always been an 11th grade course, this means that the U.S. fork in the road is one grade level later than in Singapore and Finland.

  Suggestion: If the fork can be moved so the course before Algebra II is the last common course (which is more in line with international practices), students with different interests will have time to take more specialized mathematics courses one grade sooner. Specialized courses can better prepare students for college and career-related learning.

- *Difference* — **Choices of academic programs**

In Singapore and Finland, students have choices. After the age of 16, depending on their interests, they can choose a stream or program that focuses on the academic path they wish to pursue. They do this with the guidance of the teachers, administrators and parents.

In Singapore after they make this decision, they have another two to three years in which they may transition from one stream to another before they enter a university or finalize their schooling. The streams also allow students to cover the same courses over a longer period of time.

The Singapore system allows students to focus on the level of mathematics appropriate to their career goals and circumvent having to deal with mathematics that is unnecessary or too advanced.

When they reach 16, U.S. students have two more years to attempt one of the official courses or drop out. In the U.S., 30% of students drop out of high school, many without completing even Algebra I or Geometry.

Recently in the U.S., the trend is moving away from streaming and instead requiring all students to complete Algebra II-level work in order to graduate. Making all students complete high level math such as Algebra II, regardless of educational or career goals, could be counterproductive.

If all students are required to take Algebra II, the pressures from increased enrollment by students with diverse interests and levels of preparation could negatively impact the rigor of the course. Such pressures could lead to a course too weakened for many intended STEM majors. Conversely, if the course maintains a high level of rigor, the result could be that many non-STEM majors become excluded from college opportunities.

> **Suggestion**: Consider two alternate pathways to college eligibility before Algebra II: one path leading toward Calculus for students who will pursue STEM majors and another more relevant to students who intend to major in non-STEM subject areas.

- *Difference* — **Gradations of mathematics course content**

In Singapore, mathematics course content has different gradations. Many of the more difficult mathematics topics are presented after students have been streamed into more specialized programs. For instance, in Singapore, the syllabi for the Normal Academic and Normal Technical streams are less formal than in the Special and Express streams. Essentially, the Normal Technical syllabi contain the same content as that in the Special and Express streams, except that some of the more advanced aspects have been pruned back, so that more time is available for the topics that remain. Keeping the content parallel, but appropriately scaled back, enables Normal Academic and Normal Technical students to do only the work necessary for their stream, but also retain the possibility of crossing over if they choose to pursue university enrollment.

Here is an example of the content in one topic for the Express/Special stream compared to the Normal Academic and Normal Technical stream.

Special and Express Secondary 2	**Normal Academic Secondary 3/4**	**Normal Technical Secondary 3/4**
Algebraic Manipulation **Include:** ▪ expansion of the product of algebraic expressions ▪ changing the subject of a formula ▪ finding the value of an unknown quantity in a given formula ▪ recognising and applying the special products • $(a \pm b)^2 = a^2 \pm 2ab + b^2$ • $(a - b)^2 = (a + b)(a - b)$ ▪ factorisation of algebraic expressions of the form • $a^2x^2 - b^2y^2$ • $a^2 \pm 2ab + b^2$ • $ax^2 + bx + c$ ▪ multiplication and division of simple algebraic fractions, e.g. • $\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ • $\frac{3a}{4} \div \frac{9a^2}{10}$ ▪ addition and subtraction of algebraic fractions with linear or quadratic denominator, e.g. • $\frac{1}{x-2} + \frac{2}{x-3}$ • $\frac{1}{x^2-9} + \frac{2}{x-3}$ • $\frac{1}{x-3} + \frac{2}{(x-3)^2}$	**Algebraic Manipulation** **Include:** ▪ expansion of the product of algebraic expressions ▪ changing the subject of a formula ▪ finding the value of an unknown quantity in a given formula ▪ addition and subtraction of algebraic fractions with linear or quadratic denominator, e.g. • $\frac{1}{x-2} + \frac{2}{x-3}$ • $\frac{1}{x^2-9} + \frac{2}{x-3}$ • $\frac{1}{x-3} + \frac{2}{(x-3)^2}$	**Algebraic Manipulation** **Include:** ▪ expansion of the product of two linear algebraic expressions ▪ multiplication and division of simple algebraic fractions, e.g. • $\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ • $\frac{3a}{4} \div \frac{9a^2}{10}$ ▪ changing the subject of a formula ▪ finding the value of an unknown quantity in a given formula ▪ factorisation of linear algebraic expressions of the form • $ax + ay$ (where a is a constant) • $ax + bx + kay + kby$, (where a, b, and k are constants) ▪ factorisation of quadratic expressions of the form $x^2 + px + q$ **Exclude:** ▪ use of special products • $(a \pm b)^2 = a^2 \pm 2ab + b^2$ • $(a - b)^2 = (a + b)(a - b)$ ▪ factorisation of algebraic expressions of the form • $a^2x^2 - b^2y^2$ • $a^2 \pm 2ab + b^2$ • $ax^2 + bx + c$, where $a \neq 1$ ▪ addition and subtraction of algebraic fractions

Notice that the Special and Express streams cover the same topics as the Normal Academic, only a year earlier in the curriculum. In the Normal Technical, some topics are excluded.

U.S. students should be offered mathematics courses of different gradations, based on their educational and career goals. If as mentioned previously, students were allowed to choose a path (STEM or non-STEM) earlier in high school, then they could take courses that would better meet their needs for the remaining years of high school.

The non-STEM Algebra II course could be less abstract and formal, while including more statistics, discrete mathematics, and a friendly version of the basic ideas found in calculus. In Singapore, topics from calculus are included in the course for non-STEM students, but they explicitly "exclude differentiation from first principles," that is, the treatment is informal and intuitive.

Suggestion: Consider offering more than one version of Algebra II; versions that contain content that is parallel, but at different levels of rigor.

- *Difference* — **Alignment of coursework, syllabus, and exam**

In Singapore, 75% of the 16-year-old students take either the General Certificate O Level Exam or the N(A) Exam. The N(A) Level exam is somewhat less demanding than the O Level but includes quite a bit of overlap. Some students take the GCE Normal Technical N(T) exam instead. This qualifies students for entry into Technical Education that leads to a career in the high-tech work force in 2 years, without requiring a college degree.

Across the population of teachers and students in Singapore, there is a focus on the upcoming examination, and consistent attention is paid to the syllabus. The syllabi for the different exams are highly focused and commensurate with the time available for instruction.

One factor that gets significant attention in the revision cycle of the syllabi is the practical feasibility of the previous syllabus. If there was too much to teach, the content of the new syllabus is reduced. If certain topics were too difficult, they may be moved up to a more specialized course (for example, complex numbers).

The majority of U.S. courses do not have such alignment between course objectives and examination goals. The test items are derived from a domain sampling. As a consequence, students are at a disadvantage in preparing for examinations, due to a lack of transparency of what specifically they will be tested on.

The U.S. standards are not revised based on the feasibility of the previous syllabus, as they are in Singapore.

Suggestion: Consider working off one syllabus for both coursework and exam content to maintain transparency in the expectations of the course.

Comparison of the Mathematics

Given the stated differences between the U.S. and international systems, comparison the mathematics is challenging. Careful selection of the program and courses of study within each country is important for a meaningful comparison. In order to accomplish this, the sequence of courses and topics as stated in the *Indiana Standards for Mathematics* were used as a framework to which specific topics from Finland and Singapore were correlated.

- *Difference* — **Content over time**

  In the U.S., more mathematics content is pushed into fewer courses taken in less time. In fact, Indiana (like most U.S. high schools) takes the same content Singapore spreads over 7 years and compresses it into three 1-year courses: Algebra I, Geometry and Algebra II.

  Finland also covers topics similar to as Indiana's to age 19.

> **Suggestion**: Although this is a traditional U.S. sequence, consider reviewing the amount of time given and the order of content in the high school course sequence.

Algebra I

- **Comparison Table**

Indiana 	Linear equations and inequalities	Relations and functions	Linear functions and inequalities	Pairs of linear equations and inequalities	Polynomials	Quadratic equations and functions	Data analysis
Singapore 	S1	S1	S2	S2	S2	S2	S1, S2
Finland 	Grades 6–9	Grades 6–9	Grades 6–9	Grades 6–9, Analytical geometry	Grades 6–9	Polynomial functions	Grades 6–9

Singapore

S1–S4 is comparable to U.S. grades 7–10.
H1 (non-STEM program) and H2 (STEM program) are comparable to U.S. grades 11 and 12.

Finland

Grades 6–9 are comparable to U.S. grades 6–9.
The individual course names are part of a program which is equivalent to grades 10–13.

- *Difference* — **The development of Algebra I concepts**

The introduction of basic algebra appears early in Singapore. The fundamental roots of algebra and functions are nurtured in the primary grades, thus giving students a running start toward the *Secondary Mathematics Syllabuses*. As mentioned in the K–8 section of this report, students are provided with a solid foundation in number sentences, number properties, properties of equality and quantity, thus making a smoother transition to algebra. As shown in the table on page 71, the Algebra I content is covered in grades 7–8 (S1–S2) of the Singapore syllabus.

Finland also has a solid foundation for mathematical thinking and variables. Most of the algebra content is covered before the grade 8 exam—thus in 6–8. The first course in Finland's General Upper Secondary (or U.S. grade 10) reinforces skills in dealing with equation, powers, radicals, and proportionality and examines exponential functions. In other words, the order of concepts is unlike the typical U.S. sequence of Algebra I, Geometry, Algebra II, and Pre-Calculus.

> **Suggestion**: Examine the content of the Algebra I course and the content covered in grades 7 and 8.

Algebra II

- **Comparison Table**

Indiana 	Relations and functions	Linear and absolute value equations, inequalities and functions	Matrices	Quadratic equations and functions	Polynomial equations and functions	Relational and radical expressions, equations, and functions	Exponential and logarithmic functions	Sequences and series	Data analysis and probability
Singapore 	S3/4	H1	S3/4	S3/4 complex numbers in H2	S3/4	S2	S3/4, S3/4 additional math, H1	H2	H1, H2
Finland 	Polynomial functions	Analytical geometry	Not covered	Polynomial functions	Polynomial functions	Functions and equations	Radical and logarithmic functions	Trigonometric functions and number sequences	Probability and statistics

Singapore

S1–S4 is comparable to U.S. grades 7–10.
H1 (non-STEM program) and H2 (STEM program) are comparable to U.S. grades 11 and 12.

Finland

Grades 6–9 are comparable to U.S. grades 6–9.
The individual course names are part of a program which is equivalent to grades 10–13.

- *Difference* — **Placement of complex numbers**

In Singapore, complex numbers are not taught in H2, which is the STEM program in pre-university. In Finland, complex numbers are not part of the General Upper Secondary mathematics. Although complex numbers come up as solutions to quadratic equations studied in Algebra II, they can be avoided or given minimal attention.

Indiana introduces complex numbers in Algebra II.

INDIANA

Quadratic Equations and Functions — Algebra II

A2.4.1 Define, plot, add, subtract, multiply and divide complex numbers.

Example:

Multiply $7 - 4i$ and $10 + 6i$.

A2.4.2 Solve quadratic equations in the complex number system.

Example:

Solve $x^2 - 2x + 5 = 0$ over the complex numbers.

Since Indiana Algebra II curriculum includes non-STEM students, the topic of complex numbers might be moved to a later course.

Suggestion: Move complex numbers to Pre-Calculus.

- *Difference* — **Topics covered under matrices**

The *Indiana Standards for Mathematics* for matrices in Algebra II include the topics below.

INDIANA

Matrices — Algebra II

A2.3.4 Use the properties of matrix multiplication, including identity and inverse matrices, to solve problems.

Example:

Explain how two matrices can be multiplied and what the dimensions of the product matrix will be.

(continues)

(continued)

A2.3.5 Represent a system of equations in two or three variables as a matrix equation $Ax = b$ and use technology to find $x = A^{-1}b$.

Example:

Alana's Boutique is selling faux pearls for the following prices:
2 grey faux pearls and 3 black faux pearls cost \$8.25
3 grey faux pearls and 4 black faux pearls cost \$11.25

Let x = the cost of one grey pearl.
Let y = the cost of one black pearl.

Write the system as a matrix equation. Use technology to find the cost of one grey pearl and the cost of one black pearl.

In contrast, Singapore excludes both matrix representation of geometrical transformations and solving simultaneous linear equations using the inverse matrix method.

SINGAPORE

Numbers and Algebra: Matrices — Secondary 3/4

Include:

- display of information in the form of a matrix of any order
- interpreting the data in a given matrix
- product of a scalar quantity and a matrix
- problems involving the calculation of the sum and product (where appropriate) of two matrices

Exclude:

- matrix representation of geometrical transformations
- solving simultaneous linear equations using the inverse matrix method

There is no other work with matrices for non-STEM majors to prepare for the university.

In Finland, matrices are not covered in the Upper Secondary Curriculum.

Suggestion: Consider limiting the depth of coverage for matrices in Algebra II.

- *Difference* — **Treatment of polynomials beyond quadratics**

Singapore does not study polynomials beyond quadratics with non-STEM majors.

Finland does include polynomials beyond quadratics in their compulsory courses: Functions and Equations and Polynomial functions—thus giving more emphasis to polynomial and exponential functions.

- *Difference* — **Treatment of normal distribution**

Singapore's treatment of normal distribution occurs in H1 and H2.

SINGAPORE

Normal distribution H1 and H2

Include:

- concept of a normal distribution and its mean and variance; use of N(μ, σ^2) as a probability model
- standard normal distribution
- finding the value of P($X < x_1$) given the values of x_1, μ, σ
- solving problems involving normal variables
- solving problems involving the use of E($aX + b$) and Var($aX + b$)
- solving problems involving the use of E($aX + bY$) and Var($aX + bY$), where X and Y are independent
- normal approximation to binomial

Exclude:

- finding probability density functions and distribution functions
- calculation of E(X) and Var(X) from other probability density functions

Indiana includes the standard seen below.

INDIANA

Data Analysis and Probability Algebra II

A2.9.2 Know and apply the characteristics of the normal distribution.

Suggestion: Consider defining the concepts covered under normal distribution.

- *Difference* — **Treatment of data and probability**

The Singapore H1 syllabus (non-STEM program) focuses extensively on rigorous statistics. The H1 syllabus allows time for this extensive treatment of statistics by pruning back or eliminating certain topics. For example, complex numbers are only covered in the H2 course, which is for STEM students. The treatment of calculus in H1 is pared down to an informal study of calculus suitable for students who will not be needing advanced math. Also, in the H1 syllabus, the study of functions is scaled back to the basics of linear, exponential, and logarithmic functions.

SINGAPORE

Probability H1

Include:

- addition and multiplication of probabilities
- mutually exclusive events and independent events
- use of tables of outcomes, Venn diagrams, and tree diagrams to calculate probabilities
- calculation of conditional probabilities in simple cases
- use of

$P(A') = 1 - P(A)$

$P(A \cup B) = P(A) + P(B) - P(A \cup B)$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Binomial Distribution H1

Include:

- knowledge of the binomial expansion of $(a + b)^n$ for positive integer n
- use of the notations $n!$ and $\binom{n}{r}$
- concept of binomial distribution $\mathrm{B}(n, p)$ and use of $\mathrm{B}(n, p)$ as a probability model
- use of mean and variance of a binomial distribution (without proof)
- solving problems involving binomial variables

Exclude calculation of mean and variance for other probability distributions

(continues)

(continued)

Normal Distribution H1

Include:

- concept of a normal distribution and its mean and variance;
- use of $N(\mu, \sigma^2)$ as a probability model
- standard normal distribution
- finding the value of $P(X < x_1)$ given the values of x_1, μ, σ
- use of the symmetry of the normal distribution
- finding a relationship between x_1, μ, σ given the value of $P(X < x_1)$
- solving problems involving normal variables
- solving problems involving the use of $E(aX + b)$ and $Var(aX + b)$
- solving problems involving the use of $E(aX + bY)$ and $Var(aX + bY)$, where X and Y are independent
- normal approximation to binomial

Exclude:

- finding probability density functions and distribution functions
- calculation of $E(X)$ and $Var(X)$ from other probability density functions

Sampling H1

Include:

- concepts of population and sample
- random, stratified, systematic and quota samples
- advantages and disadvantages of the various sampling methods
- distribution of sample means from a normal population
- use of the Central Limit Theorem to treat sample means as having normal distribution when the sample size is sufficiently large
- calculation of unbiased estimates of the population mean and variance from a sample
- solving problems involving the sampling distribution

Hypothesis Testing H1

Include:

- concepts of null and alternative hypotheses, test statistic level of significance and p-value
- tests for a population mean based on:
  - a sample from a normal population of known variance
  - a large sample from any population
- 1-tail and 2-tail tests

Exclude testing the difference between two population means

(continues)

(continued)

Correlation Coefficient and Linear Regression H1

Include:

- concepts of scatter diagram, correlation coefficient and linear regression
- calculation and interpretation of the product moment correlation coefficient and of the equation of the least squares regression line
- concepts of interpolation and extrapolation

Exclude:

- derivation of formulae
- hypothesis tests
- use of a square, reciprocal or logarithmic transformation to achieve linearity

STEM students, after the fork, begin a rigorous two-year study of functions and analysis, which includes content comparable to that in Indiana's pre-calculus, AP calculus, and an introduction to differential equations. It includes composition, decomposition, inverse functions, series, vectors, complex numbers, and other topics in calculus. STEM students also study the rigorous statistics included in the H1 syllabus.

Indiana covers these topics in data and probability.

INDIANA

Data Analysis and Probability Algebra II

A2.9.1 Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.

A2.9.2 Know and apply the characteristics of the normal distribution.

A2.9.3 Use permutations, combinations, and other counting methods to determine the number of ways that events can occur and to calculate probabilities including the probability of compound events.

Suggestion: Consider offering more data and probability in a non-STEM major version of Algebra II.

Geometry

Comparison Table

Indiana	Points, lines, angles and planes	Polygons	Triangles	Right triangles	Circles	Polyhedra and other solids	Geometric proof, reasoning and communication
Singapore	S1	S1 congruence and similarity in S2	S1 congruence and similarity in S3/4	S2 trigonometry in S3/4	S3/4	S1, S2	S3/4 additional math
Finland	Analytical geometry, Grades 6–9	Grades 6–9	Grades 6–9	Grades 6–9, Geometry	Grades 6–9, Geometry	Grades 6–9, Geometry	Reasoning throughout—proof not in compulsory

Singapore

S1–S4 is comparable to U.S. grades 7–10.
H1 (non-STEM program) and H2 (STEM program) are comparable to U.S. grades 11 and 12.

Finland

Grades 6–9 are comparable to U.S. grades 6–9.
The individual course names are part of a program which is equivalent to grades 10–13.

Difference — Content in geometry courses

Finland covers many of the topics in Indiana's Geometry course in grades 6–9. After grade 9, Finland has two 32–day courses called Analytic Geometry and Geometry.

FINLAND

Geometry MAA3

OBJECTIVES

The objectives of the course are for students to

- gain practice in perceiving and describing information about space and shape in both two and three dimensions;
- gain practice in formulating, justifying and using statements dealing with geometrical information;
- solve geometrical problems, making use of the properties and similarity of figures and solid bodies, Pythagoras' theorem and the trigonometry of right-angled and oblique triangles.

CORE CONTENTS

- similarity of figures and bodies;
- sine and cosine rules;
- geometry of a circle, its parts and straight lines related to it;
- calculating lengths, angles, areas and volumes related to figures and bodies.

(continues)

(continued)

Analytical geometry MAA4

OBJECTIVES

The objectives of the course are for students to

- understand how analytical geometry links geometric and algebraic concepts;
- understand the concept of the equation of a set of points and learn to examine points, straight lines, circles and parabolas using equations;
- consolidate their understanding of the concept of absolute value and learn to solve absolute value equations and corresponding inequalities of the form $|f(x)| = a$ or $|f(x)| = |g(x)|$;
- reinforce their skills in solving systems of equations.

CORE CONTENTS

- equations of sets of points;
- equations of straight lines, circles and parabolas;
- solving absolute value equations and inequalities;
- solving equation systems;
- distance of a point from a straight line.

All the Indiana Geometry content is covered in Singapore Secondary 1–4 (grades 7–10). The Singapore H1 (non-STEM majors) syllabus does not have a geometry component. The H2 (STEM majors) syllabus contains vectors and 3-dimensional geometry.

Pre-Calculus

• Comparison Table

Indiana	Relations and functions	Conics	Logarithmic and exponential functions	Trigonometry in triangles	Trigonometric functions	Trigonometric identities and equations	Polar coordinates and complex numbers	Sequences and series	Vectors and parametric equations	Data analysis
Singapore	H1, H2	Non-existent	S3/4, H1	S3/4	S3/4 additional math, H1	S3/4 additional math	H2	H2	S3/4 (two dimensions), H2 (two and three dimensions)	H1, H2
Finland	Functions and equations	Not mentioned	Radical and logarithmic functions	Trigonometric functions and number sequences	Trigonometric functions and number sequences	Trigonometric functions and number sequences	Not mentioned	Trigonometric functions and number sequences	Vectors	Not mentioned

Singapore

S1–S4 is comparable to U.S. grades 7–10.
H1 (non-STEM program) and H2 (STEM program) are comparable to U.S. grades 11 and 12.

Finland

Grades 6–9 are comparable to U.S. grades 6–9.
The individual course names are part of a program which is equivalent to grades 10–13.

- *Difference* — **Content in Pre-Calculus course**

  Most of the content in the Pre-Calculus course is covered in Singapore Secondary 3–4 or H1 and H2. Trigonometric equations, polar coordinates, complex numbers, sequences and series, three-dimensional vectors are only required for Singapore STEM majors. Conics is not part of the high school curriculum in Singapore.

  In Finland, conic sections and polar coordinates are not dealt with in Upper Secondary.

Mathematics Notation

Each Singapore syllabus contains a list of mathematics notations that students should master. The H1 syllabus has a 5-page list of rotation. An excerpt is below.

SINGAPORE

Mathematics Notation H1

f	function f
$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f}: A \rightarrow B$	f is a function under which each element of set A has an image in set B
$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim\limits_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx; δx	an increment of x
$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{(n)}(x)$	the first, second, …, nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y\,\mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y\,\mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\dfrac{\partial y}{\partial x}$	the partial derivative of y with respect to x
$\dot{x}, \ddot{x}, \ldots$	the first, second, . . . derivatives of x with respect to time

Design & Technology Syllabus

Lower Secondary
Special/Express/Normal (Academic)

Ministry of Education
SINGAPORE

Year of implementation: from 2007

CONTENTS

TABLES

INTRODUCTION

Design & Technology (D&T) is part of a holistic broad-based education. It is a compulsory project-based subject in the lower secondary school curriculum. D&T anchors on design action and the application of knowledge and process skills.

AIMS

The lower secondary D&T syllabus aims to enable pupils to:

- develop an awareness of design in the made-world;
- develop an appreciation of function, aesthetics and technology in design;
- develop basic design thinking and communication skills;
- experience the process of realising design through making; and
- think and intervene creatively to become autonomous decision makers.

Pupils are to engage in design-and-make activities and experience a basic process of design adapted to their abilities, interest and design context.

SYLLABUS FRAMEWORK

The D&T syllabus comprises **three learning domains** broadly classified as **Design Appreciation**, **Designing** and **Making**. Pupils will learn to apply knowledge and skills through design-and-make activities.

Design Appreciation
Design appreciation exposes pupils to product awareness, product functions, aesthetics, design in society, and cultural and technological influences on design.

Designing
Through the design process, pupils learn and practise basic strategies and skills in research, analysis, ideation and development, evaluation and design communication.

Making
Pupils acquire basic making skills and related knowledge through the manipulation of resistant materials, simple modelling materials and basic technology; and the execution of appropriate techniques and processes in a workshop environment. Through making, pupils also learn to use materials prudently and develop the habit of effective and efficient work processes in a safe manner.

SUBJECT CONTENT

The topics for each of the learning domains are listed in Table 1. These topics are suggested content to be taught in the context of the design-and-make activities. Teaching of the topics should take an integrative approach to help pupils appreciate the application of related knowledge and skills. Pupils are encouraged to draw on knowledge and understanding from other subject areas like Science, Mathematics and Art.

Table 1 Summary of Topics

Design Appreciation	Designing	Making
1 Aesthetics 2 Design in Society 3 Sustainability 4 Basic Technology	5 Design Method 6 Need Definition 7 Research 8 Idea Generation and Development 9 Communication 10 Evaluation	11 Planning 12 Materials 13 Practical Processes

Domain 1 Design Appreciation

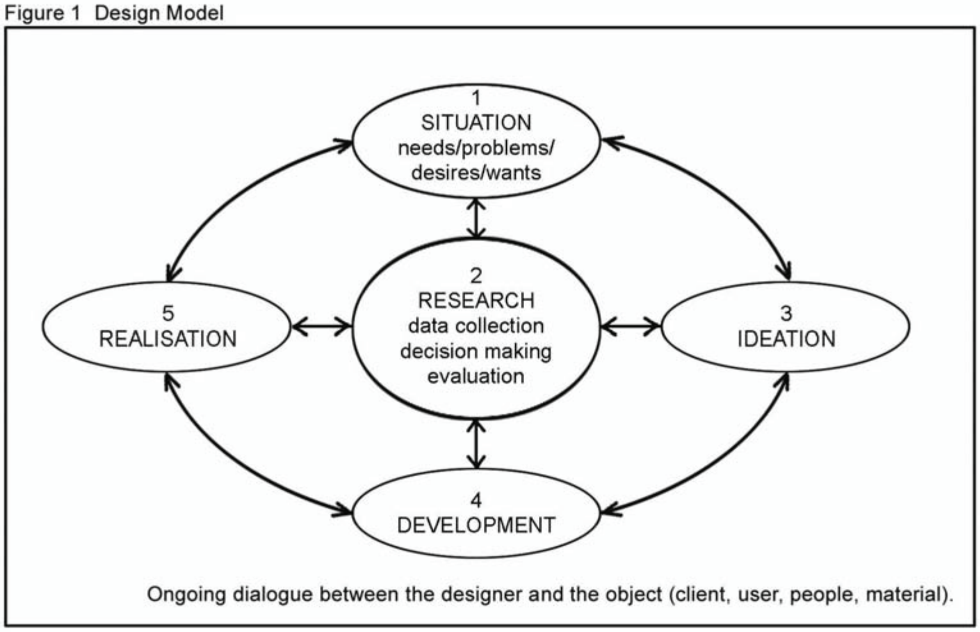
Topic	Pupils should be able to:
1 Aesthetics	▪ appreciate the role of line, shape & form, colour, texture, proportion and ergonomics in relation to design needs ▪ understand the link between ergonomics, function and aesthetics
2 Design in Society	▪ show an awareness of societal and cultural influences on design
3 Sustainability	▪ demonstrate awareness of environmental considerations related to materials usage, production methods and after-use disposal
4 Basic Technology ▪ Electronics ▪ Mechanisms ▪ Structures	▪ appreciate how technology is applied to enhance product functionality ▪ understand the roles electronics play in everyday life ▪ demonstrate awareness in circuit connection and the use of common electronic components ▪ show awareness of the working principles of simple machines and mechanisms in familiar products ▪ describe the use of mechanisms in conversion and transmission of motion ▪ appreciate how structures contribute to strength and aesthetic appeal ▪ understand what a structure is and the need for structures ▪ recognise the use of different methods of reinforcing structures

Domain 2 Designing

5 Design Method

A convenient model to help pupils engage in design activity is shown in Figure 1.

Figure 1 Design Model



Note: The numbering of the design stages is meant as a guide.

The arrows show that design is not always a linear process but is dynamic in nature, requiring looping back to other stages of the design model. For example, in the process of realisation, there may be further development to the design solution. The numbered steps serve as a guide on the sequence of design activities pupils would generally undertake in a design-and-make project.

Domain 2 Designing (cont'd)

Topic	Pupils should be able to:
6 Need Definition ▪ Analysing ▪ Image Board ▪ Design Factors	▪ define a need by considering - the needs and values of intended users - factors that affect design e.g. function, aesthetics ▪ write a design brief ▪ write specifications
7 Research ▪ Product Analysis	▪ appreciate the need for relevant information to make sound design decisions ▪ gather relevant information that will help in their designing ▪ investigate and evaluate a range of relevant consumer products in terms of meeting needs and fitness for purpose
8 Idea Generation and Development	▪ use SCAMPER or other ideation techniques to generate ideas ▪ develop ideas by considering design factors ▪ use models or mock-ups to test ideas
9 Communication ▪ Freehand Sketching ▪ Working Drawings ▪ Design Modelling	▪ express design ideas using - freehand sketching techniques with meaningful annotations - models and mock-ups ▪ prepare working drawings
10 Evaluation	▪ reflect on the progress of their work as they design and make ▪ evaluate their artefact against design specifications ▪ identify ways they could improve their artefact

Domain 3 Making

Topic	Pupils should be able to:
11 Planning	▪ prepare an outline plan of the steps in making, including the materials to be used
12 Materials ▪ Resistant Materials (wood, metal & plastics) ▪ Modelling Materials	▪ understand the basic properties of common materials in relation to their use ▪ manipulate a range of materials using suitable practical processes to model and realise design ideas
13 Practical Processes ▪ Measuring and Marking Out ▪ Holding, Cutting and Shaping ▪ Bending and Forming ▪ Joining and Assembling ▪ Finishing	▪ demonstrate the correct use of tools and machines ▪ measure, mark out, cut and shape a range of materials using appropriate techniques ▪ use jigs and formers to bend or form materials ▪ use appropriate methods to join parts of a job to form the desired structure or give the required movement ▪ use finishes to enhance the artefact function and appearance

ASSESSMENT GUIDELINES

Assessment Objectives

The following assessment objectives for each learning domain are designed to reflect the intent of the syllabus.

Pupils should be able to:

Design Appreciation

1 demonstrate awareness of societal and technological influences in design;

2 demonstrate the ability to apply knowledge in design, materials, processes and basic technology;

Designing

3 define a need by considering appropriate human, functional and aesthetic factors;

4 gather and use relevant information for design decision making;

5 generate and develop ideas using appropriate methods;

6 test and evaluate their design ideas, making appropriate modifications;

7 apply appropriate communication techniques to inform and defend ideas;

Making

8 plan the steps in making their artefact;

9 realise their artefact in appropriate material(s) using suitable techniques; and

10 make appropriate modifications to enhance the artefact.

Assessment Mode

Pupils will be assessed through purposefully designed projects which require demonstration of learning outcomes across the three learning domains. All projects shall include evidence of pupils' learning in the form of documentation of the design process and artefacts/models/mock-ups. Schools may include presentation boards for assessment.

Pupils may also be assessed through written examination. Schools have the autonomy on the design of the written paper format.

The weighting of the two assessment modes is as follows:

Assessment Mode	Weighting
Project	80% (minimum)
Written Examination	20% (maximum)

Assessment Grid

Table 2 lists the criteria for each learning domain upon which a design-and-make project is assessed.

The three learning domains are weighted to give an indication of their relative importance. They are not intended to provide a precise statement of the number of marks allocated to particular assessment objectives or criteria.

The suggested maximum mark provides a guide on the relative emphasis given to each assessment criterion.

Table 2 Assessment Grid

Learning Domain	Assessment Criteria	Suggested Maximum Mark	Weighting
Design Appreciation	Design Awareness	20	30%
	Knowledge Application	40	
Designing	Needs Analysis	10	40%
	Research	15	
	Idea Generation and Development	25	
	Communication	20	
	Evaluation	10	
Making	Planning	10	30%
	Making	50	
		Total	100%

Table 3 (from Pages 10 and 11) shows the assessment rubric for each criterion. Not every criterion may be assessed in a single design-and-make project. Any adjustments made to the suggested maximum mark shall maintain the relative emphasis of the criteria within each domain, and the weighting of the three domains. An example to illustrate the assessment of a design-and-make project is given in Table 4 on page 12.

To cover all criteria, pupils are to be given a range of projects with emphasis on different aspects of the learning domains. The marks given to each project would be added to give the total marks for the programme. Schools may decide on the weighting of each project depending on the extent of the project. An example to illustrate the computation of the total marks is given in Table 5 on page 13.

As a guide, 5 - 8 projects may be planned for the lower secondary programme (Sec 1 and 2). Each project may take 4 - 6 weeks.

Table 3 Assessment Rubric

Criteria	Level 1	Level 2	Level 3	Level 4	Level 5
DESIGN APPRECIATION					
Design Awareness	Recognises aesthetic appeal of familiar products	Recognises cultural and technological influences on designed products	Demonstrates some judgement on design aesthetics and functionality	Demonstrates ability to evaluate quality of designed products	Demonstrates ability to evaluate impact of designed products on society and environment
Knowledge Application	Recognises characteristics of familiar products	Generates ideas based on own experience of working with materials	Uses information gathered from research and knowledge of familiar products	Uses relevant information gathered, showing understanding of form and function of familiar products when developing ideas	Makes connections with other subject areas, considering cultural and environmental issues when developing ideas
DESIGNING					
Needs Analysis	Need stated with little awareness of main requirements	Need stated with some relevant design requirements identified	Need defined with relevant design requirements identified	Need clearly defined with some investigation into design requirements	Need well analysed with considerations on user, function and aesthetics
Research	Some information gathered from research	Some research with relevant information identified	Some research with use of information gathered for decision making	Adequate research with relevant information gathered and used for decision making	Thorough research with analysis of relevant information for decision making
Idea Generation and Development	Generates ideas by drawing on own experience of familiar products	Generates own ideas with some evidence of refinement	Generates and develops ideas, recognising the basic needs of the design	Generates and develops ideas with considerations on some requirements key to the needs identified	Generates imaginative ideas; develops ideas in detail with considerations on user needs, function and appearance

Table 3 Assessment Rubric (cont'd)

Criteria	Level 1	Level 2	Level 3	Level 4	Level 5
DESIGNING (cont'd)					
Evaluation	Recognises how well the artefact serves its function	Recognises strengths and areas of improvement of the artefact	Modifies artefact towards improvement during making	Tests and modifies mock-up/artefact, showing understanding of design requirements	Makes modifications on design solution via on-going evaluation and testing against specifications
Communication	Describes ideas using pictures and words	Describes information in some detail with sketches and notes	Communicates clearly with annotated sketches and/or models	Communicates clearly with sufficient detail and appropriate use of graphics and models	Communicates effectively with detailed competent graphic presentation and models
MAKING					
Planning	Needs some guidance to plan steps in making	Able to plan main steps in making with minimal guidance	Able to plan main steps in making, showing some awareness of processes	Able to plan steps in making with some details on construction process	Able to monitor own progress with detailed planning of steps in making
Making	Some competency demonstrated	Fairly well managed artefact	Fairly well managed artefact; effective solution	Well managed artefact; effective solution	Well managed and well finished artefact; desirable solution

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Table 4 Example of Assessment of a Design-and-Make Project

Criteria	Level 1	Level 2	Level 3	Level 4	Level 5	Mark Awarded	Subtotal x Weighting	Weighted Mark
DESIGN APPRECIATION								
Knowledge Application	1 - 8 Recognises characteristics of familiar products	9 - 16 Generates ideas based on own experience of working with materials	17 - 24 Uses information gathered from research and knowledge of familiar products	25 - 32 Uses relevant information gathered, showing understanding of form and function of familiar products when developing ideas	33 - 40 Makes connections with other subject areas, considering cultural and environmental issues when developing ideas	20	20/40 x 30%	15
DESIGNING								
Idea Generation and Development	1 - 5 Generates ideas by drawing on own experience of familiar products	6 - 10 Generates own ideas with some evidence of refinement	11 - 15 Generates and develops ideas, recognising the basic needs of the design	16 - 20 Generates and develops ideas with considerations on some requirements key to the needs identified	21 - 25 Generates imaginative ideas; develops ideas in detail with considerations on user needs, function and appearance	18	30/45 x 40%	27
Communication	1 - 4 Describes ideas using pictures and words	5 - 8 Describes information in some detail with sketches and notes	9 - 12 Communicates clearly with annotated sketches and/or models	13 - 16 Communicates clearly with sufficient detail and appropriate use of graphics and models	17 - 20 Communicates effectively with detailed competent graphic presentation and models	12		
MAKING								
Making	1 - 10 Some competency demonstrated	11 - 20 Fairly well managed artefact	21 - 30 Fairly well managed artefact; effective solution	31 - 40 Well managed artefact; effective solution	41 - 50 Well managed and well finished artefact; desirable solution	45	45/50 x 30%	27
							Total	69

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Table 5 Example of Computation of Total Mark at the End of Secondary One

Criteria / Project No.	Design Awareness	Knowledge Application	Needs Analysis	Research	Idea Generation and Development	Evaluation	Communication	Planning	Making	Total Marks / 100	Weighting (%)	Weighted Total (Marks × Weighting)
1		✓			✓		✓		✓	69	10	6.9
2	✓			✓		✓	✓		✓	55	15	8.3
3		✓	✓	✓	✓			✓	✓	83	25	20.1
4	✓	✓	✓	✓	✓	✓	✓	✓	✓	60	50	30.0
Total Mark for Programme											100	65.3

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